We choose to graph f with $a = 1$. Then

$$f(x) = (x + 3)(x - 2)(x - 5) = x^3 - 4x^2 - 11x + 30$$

Figure 10 shows the graph of f. Notice that the x-intercepts are -3, 2, and 5.

STEPS

1 – Determine End behavior = x^3

2—Determine the "y" intercept. = (0,30)

Determine the "x" intercepts = (-3,0),(2,0),(5,0)

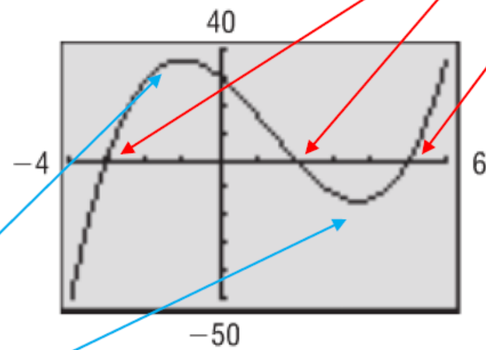
3-- Determine multiplicity

"r" or x intercepts are odd so they cross the x axis.

4—**Determine turning points degree = n → n-1**

n =3 3-1 = 2 turning points

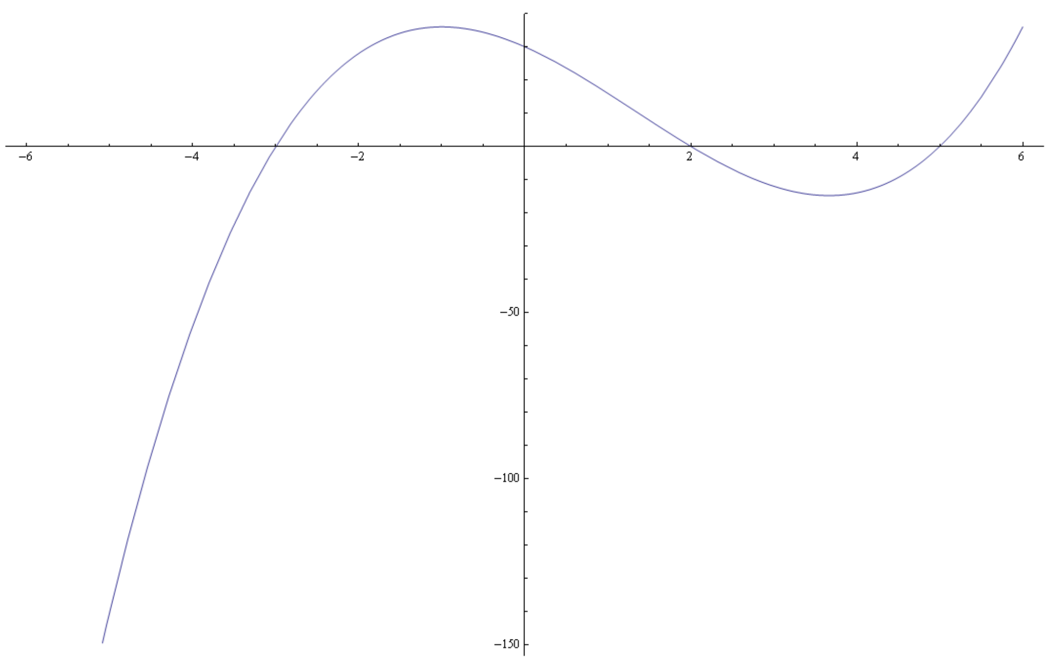
5—Put it all together



Untitled-1 *

```
In[4]:= Plot[x^3 - 4 x^2 - 11 x + 30, {x, -6, 6}]
Solve[x^3 - 4 x^2 - 11 x + 30 == 0]
```

Out[4]=



Out[5]= $\{\{x \to -3\}, \{x \to 2\}, \{x \to 5\}\}$

130%

Use the Rational Zeros Theorem to List the Potential Rational Zeros of a Polynomial Function

The next result, called the **Rational Zeros Theorem,** provides information about the rational zeros of a polynomial *with integer coefficients*.

Rational Zeros Theorem

Let f be a polynomial function of degree 1 or higher of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad a_n \neq 0, \quad a_0 \neq 0$$

where each coefficient is an integer. If $\frac{p}{q}$, in lowest terms, is a rational zero of f, then p must be a factor of a_0 and q must be a factor of a_n.

SUMMARY Steps for Finding the Real Zeros of a Polynomial Function

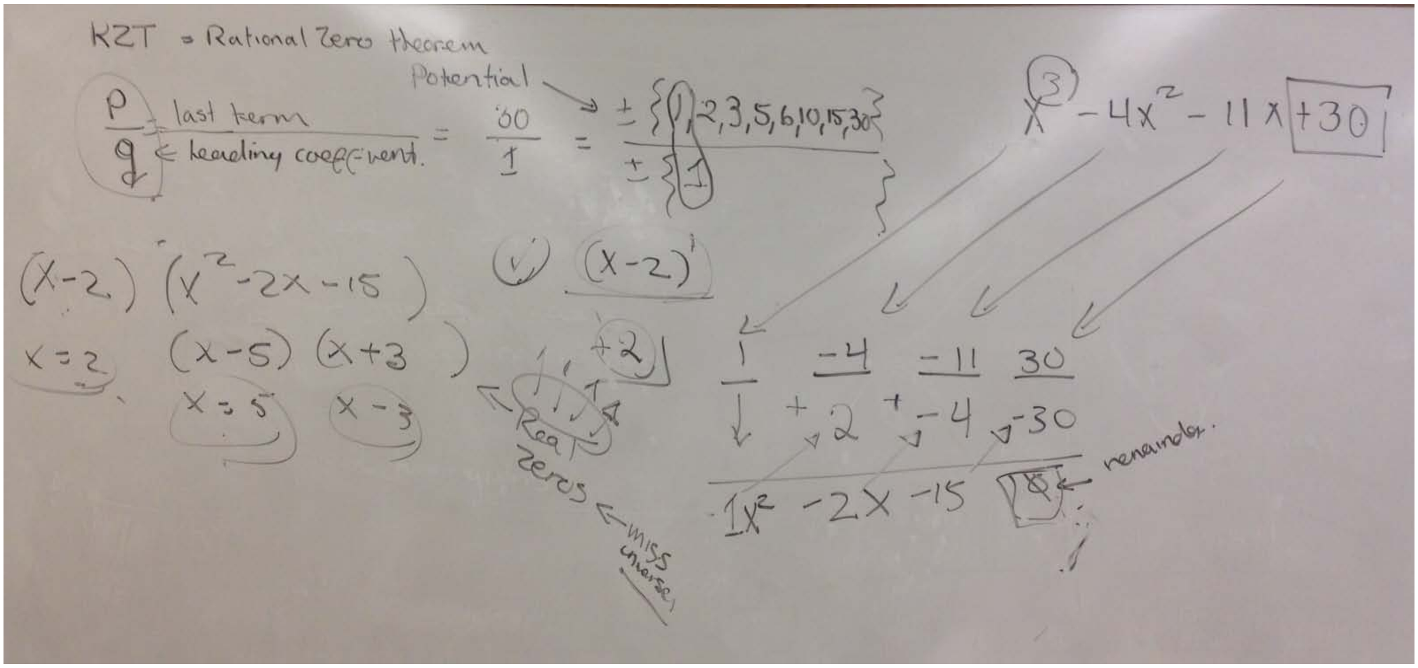
STEP 1: Use the degree of the polynomial to determine the maximum number of real zeros.

STEP 2: (a) If the polynomial has integer coefficients, use the Rational Zeros Theorem to identify those rational numbers that potentially could be zeros.

(b) Use substitution, synthetic division, or long division to test each potential rational zero. Each time that a zero (and thus a factor) is found, repeat Step 2 on the depressed equation.

In attempting to find the zeros, remember to use (if possible) the factoring techniques that you already know (special products, factoring by grouping, and so on).

Special products	Factoring by groups
SPECIAL PRODUCT PATTERNS **Sum and Difference** — **Example** $(a+b)(a-b) = a^2 - b^2$ — $(x+6)(x-6) = x^2 - 36$ **Square of a Binomial** — **Example** $(a+b)^2 = a^2 + 2ab + b^2$ — $(x+5)^2 = x^2 + 10x + 25$ $(a-b)^2 = a^2 - 2ab + b^2$ — $(2x-3)^2 = 4x^2 - 12x + 9$ **Cube of a Binomial** — **Example** $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ — $(x+3)^3 = x^3 + 9x^2 + 27x + 27$ $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ — $(x-4)^3 = x^3 - 12x^2 + 48x - 64$	**Factor the polynomial $x^3 - 3x^2 - 36x + 108$.** **Solution** $x^3 - 3x^2 - 36x + 108$ $= (x^3 - 3x^2) + (-36x + 108)$ **Group terms.** $= x^2(x - 3) + (-36)(x - 3)$ **Factor each group.** $= (x^2 - 36)(x - 3)$ **Distributive property** $= (x + 6)(x - 6)(x - 3)$ **Difference of two squares**


RZT = Rational Zero theorem
Potential
p / q ← last term / leading coefficient = 30 / 1 = ± {1, 2, 3, 5, 6, 10, 15, 30} / ± {1}
x³ − 4x² − 11x +30
(x−2) (x² −2x −15)
x = 2 (x−5) (x+3)
x = 5 x −3
(x−2)
+2 | 1 −4 −11 30
↓ 2 −4 −30
1x² −2x −15 0 ← remainder
Real Zeros
← miss universe