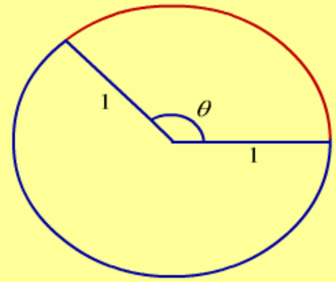
6.1 Radian and Degree Measure of Angles

Radian Measure

Let θ (the Greek letter *theta*) be an angle at the center of a circle of radius 1, as shown in the diagram. The measure of θ in **radians** (abbreviated as **rad**) is the length of that portion of the circle subtended by θ (that is,the portion of the circumference shown in red). Note that the unit of length measurement is immaterial. As long as the circle has a radius of 1 (unit), the length of the subtended portion of the circle (in the same units) is defined to be the radian measure of the angle.



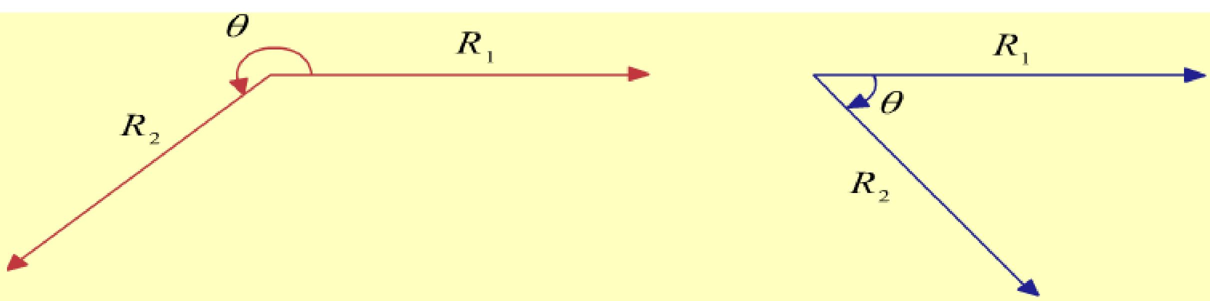
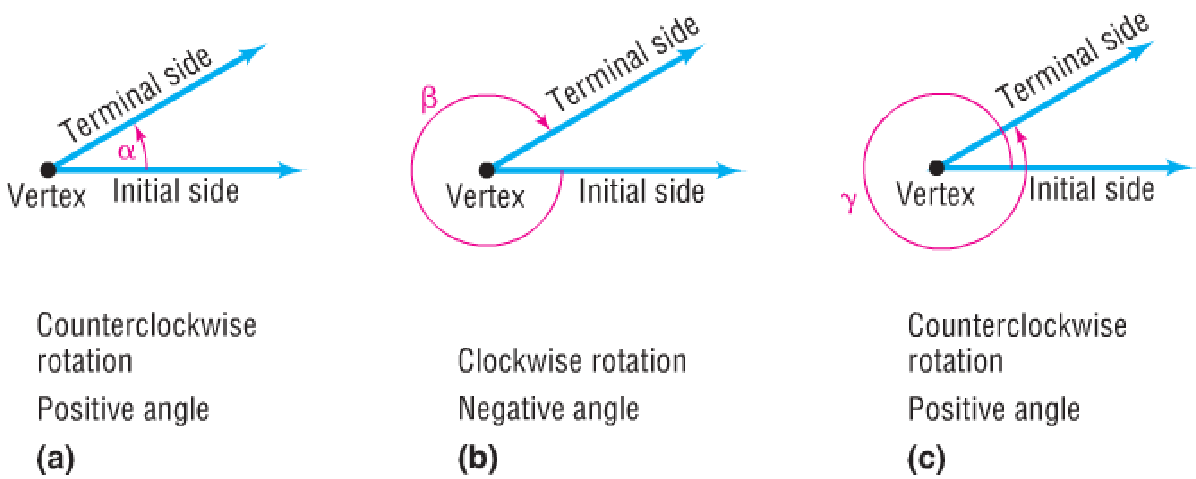
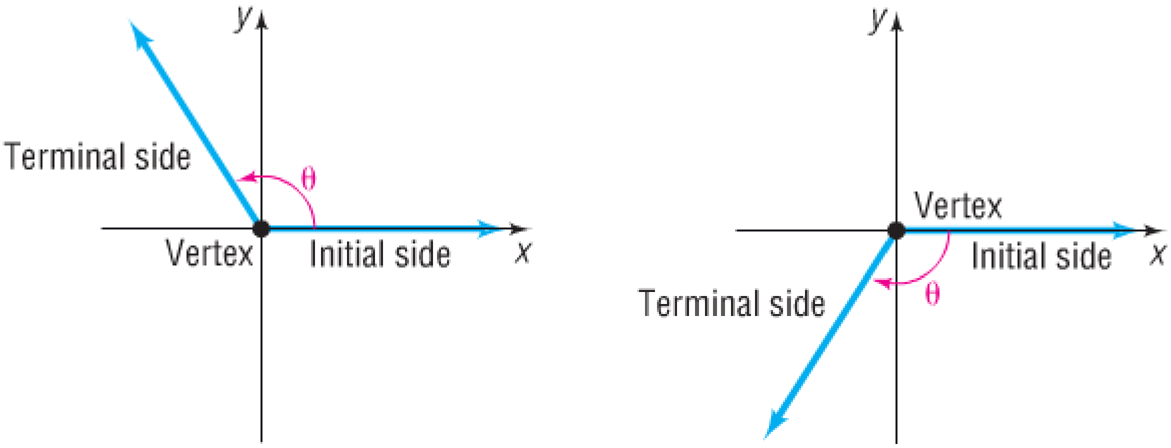


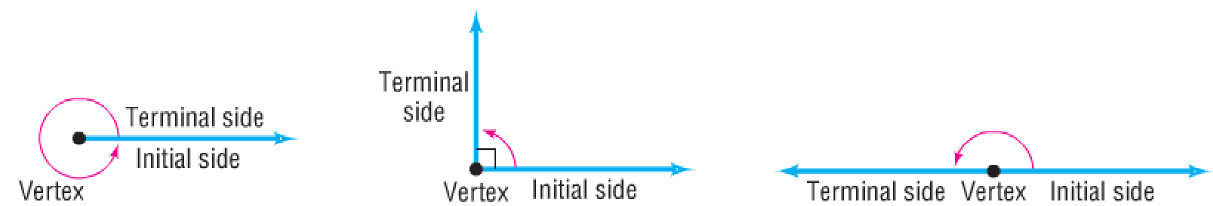
Figure 1: Positive and Negative Angle Measure





(a) θ is in standard position; θ is positive

(b) θ is in standard position; θ is negative



(a) 1 revolution counterclockwise, 360°

(b) right angle, $\frac{1}{4}$ revolution counterclockwise, 90°

(c) straight angle, $\frac{1}{2}$ revolution counterclockwise, 180°

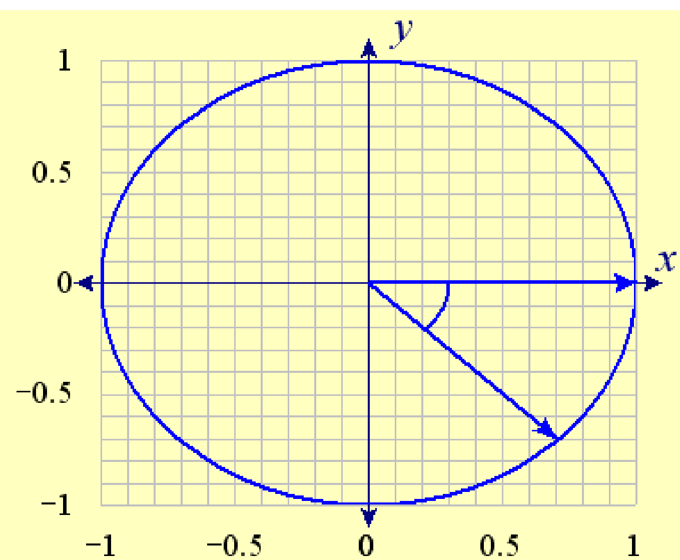


Figure 2: Standard Position of an Angle

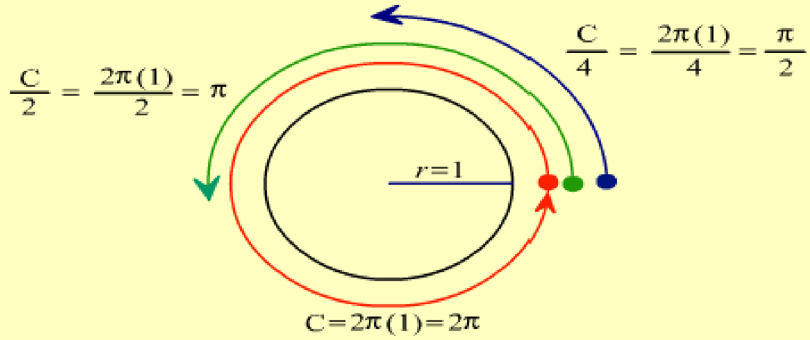


Figure 3

Conversion Formulas

Since $180° = \pi$ rad, we know that $1° = \frac{\pi}{180}$ rad and $\left(\frac{180}{\pi}\right)^{\circ} = 1$ rad.

Multiplying both sides of these equations by an arbitrary quantity x, we have:

1. $x° = (x)\left(\frac{\pi}{180}\right)$rad, and
2. x rad $= (x)\left(\frac{180}{\pi}\right)^{\circ}$.

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees} \qquad \textbf{(7)}$$

Convert each angle in degrees to radians.

(a) $60°$ (b) $150°$ (c) $-45°$ (d) $90°$ (e) $107°$

(a) $60° = 60 \cdot 1 \text{ degree} = 60 \cdot \frac{\pi}{180} \text{ radian} = \frac{\pi}{3} \text{ radians}$

(b) $150° = 150 \cdot 1° = 150 \cdot \frac{\pi}{180} \text{ radian} = \frac{5\pi}{6} \text{ radians}$

(c) $-45° = -45 \cdot \frac{\pi}{180} \text{ radian} = -\frac{\pi}{4} \text{ radian}$

(d) $90° = 90 \cdot \frac{\pi}{180} \text{ radian} = \frac{\pi}{2} \text{ radians}$

(e) $107° = 107 \cdot \frac{\pi}{180} \text{ radian} \approx 1.868 \text{ radians}$

Convert each angle in radians to degrees.

(a) $\frac{\pi}{6}$ radian (b) $\frac{3\pi}{2}$ radians (c) $-\frac{3\pi}{4}$ radians

(d) $\frac{7\pi}{3}$ radians (e) 3 radians

(a) $\frac{\pi}{6}$ radian $= \frac{\pi}{6} \cdot 1$ radian $= \frac{\pi}{6} \cdot \frac{180}{\pi}$ degrees $= 30°$

(b) $\frac{3\pi}{2}$ radians $= \frac{3\pi}{2} \cdot \frac{180}{\pi}$ degrees $= 270°$

(c) $-\frac{3\pi}{4}$ radians $= -\frac{3\pi}{4} \cdot \frac{180}{\pi}$ degrees $= -135°$

(d) $\frac{7\pi}{3}$ radians $= \frac{7\pi}{3} \cdot \frac{180}{\pi}$ degrees $= 420°$

(e) 3 radians $= 3 \cdot \frac{180}{\pi}$ degrees $\approx 171.89°$

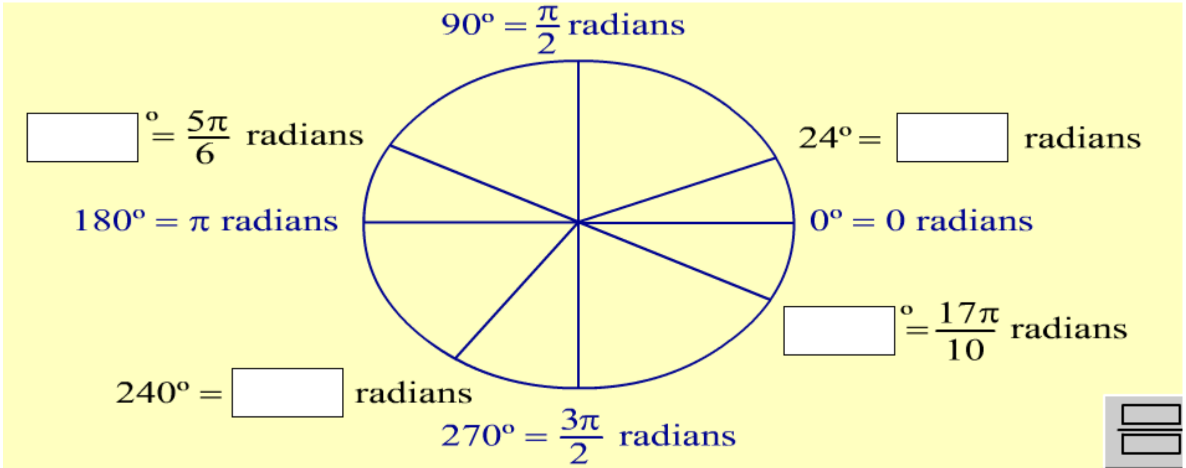
Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

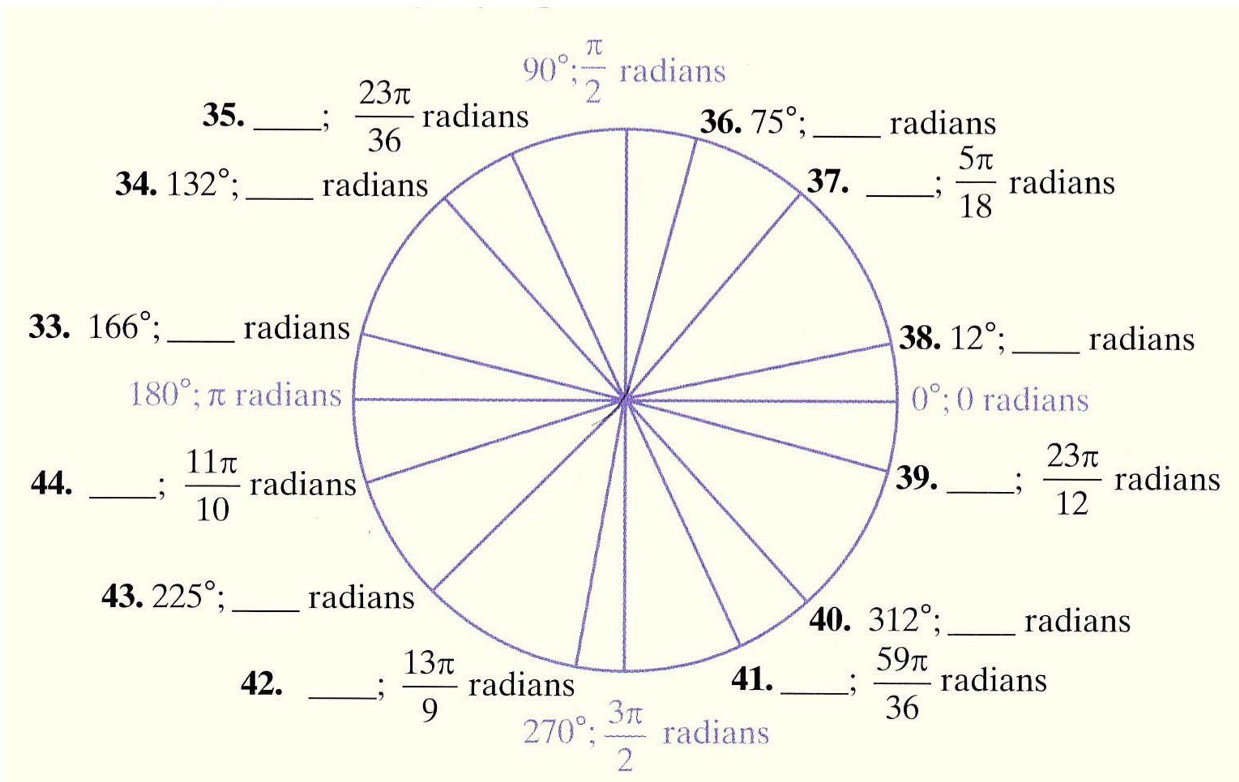
53. Convert the radian measure to degrees, or the degree measure to radians.

(a) $\frac{3\pi}{2}$ (b) 630° (c) $\frac{11\pi}{6}$ (d) 270°

~DO CONVERSION PROBLEMS~

UNIT CIRCLE



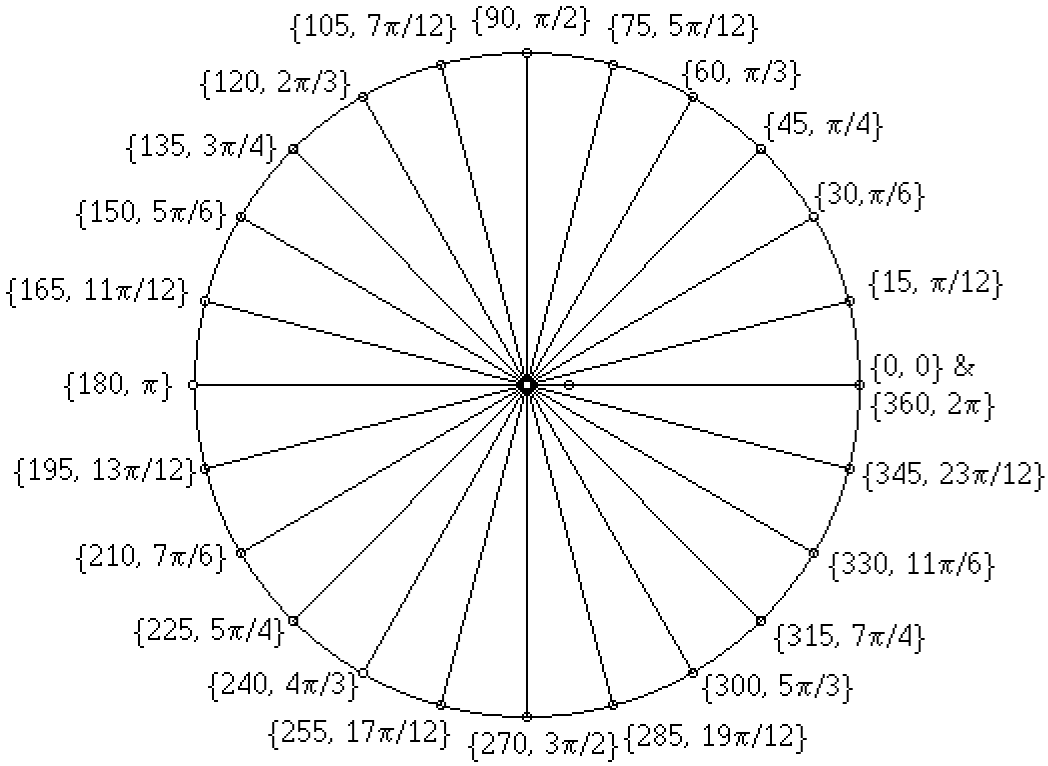


$$30° = (\ 30\ )\left(\frac{\pi}{180}\right) =$$

$$45° = (\ 45\ )\left(\frac{\pi}{180}\right) =$$

$$60° = (\ 60\ )\left(\frac{\pi}{180}\right) =$$

$$90° = (\ 90\ )\left(\frac{\pi}{180}\right) =$$



Quadrant II Angles between $\pi/2$ and π x is negative and y is positive	Quadrant I Angles between 0 and $\pi/2$ x is positive and y is positive
Quadrant III Angles between π and $3\pi/2$ x is negative and y is negative	Quadrant IV Angles between $3\pi/2$ and 2π x is positive and y is negative

TRIANGLES

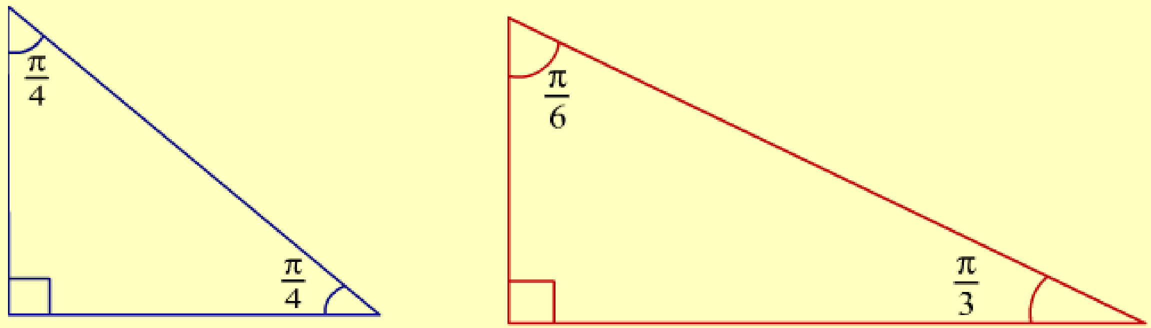


Figure 4: Common Triangles

Recall also that the sum of the angles of a triangle is always 180°, or π radians.

Pythagorean Theorem $a^2 + b^2 = c^2$

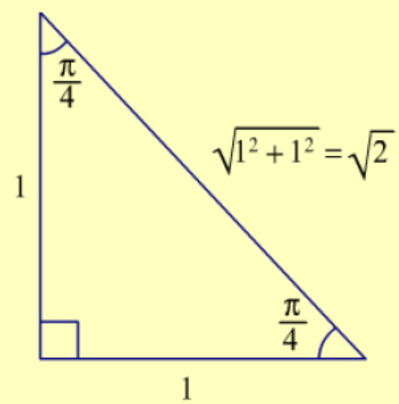
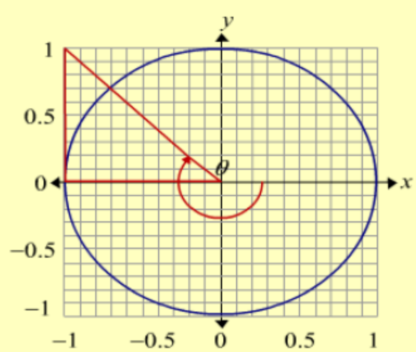


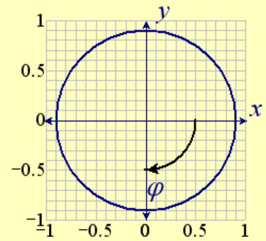
Figure 5

Use the information in each diagram to determine the radian measure of the indicated angle.

a.



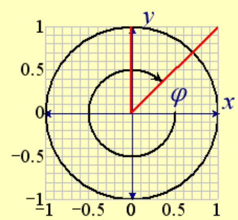
You were asked to use the information in the diagram below to determine the radian measure of the angle φ.



This angle is in standard position and its measure is negative. It is defined by beginning at the positive x-axis and making $\frac{1}{4}$ of a revolution. Since the angle is rotating in the negative, we know that one full revolution is equivalent to -2π radians, so one–fourths of a revolution is $\left(\frac{1}{4}\right)(-2\pi) = \frac{-\pi}{2}$ radians. Hence, this angle has a measure of $\frac{-\pi}{2}$.

Answer: $\frac{-\pi}{2}$

You were asked to use the information in the diagram below to determine the radian measure of the angle φ.



This angle is in standard position and its measure is negative. It is defined by beginning at the positive x-axis and rotating $\frac{-3\pi}{2}$ radians and then a little bit more. The "bit more" comes from the angle whose initial side is the positive y-axis and whose terminal side contains the hypotenuse of the red $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ triangle. We know that it is this type of triangle because the lengths of the two legs of the right triangle are equivalent. So the "bit more" is $\frac{-\pi}{4}$ radians, and altogether the angle φ has a measure of $-\frac{3\pi}{2} - \frac{\pi}{4} = \frac{-7\pi}{4}$.

Answer: $\frac{-7\pi}{4}$

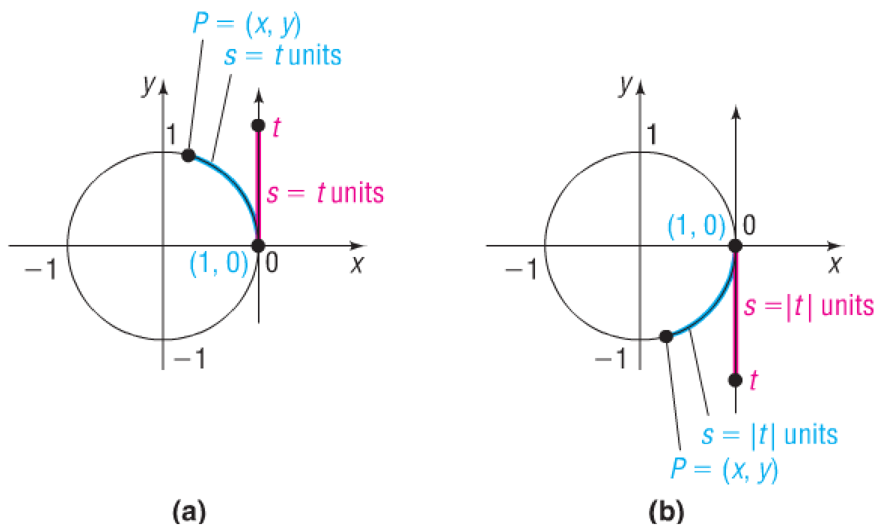
Let t be any real number. We position the t-axis so that it is vertical with the positive direction up. We place this t-axis in the xy-plane so that $t = 0$ is located at the point $(1, 0)$ in the xy-plane.

If $t \geq 0$, let s be the distance from the origin to t on the t-axis. See the red portion of Figure 18(a).

Now look at the unit circle in Figure 18(a). Beginning at the point $(1, 0)$ on the unit circle, travel $s = t$ units in the counterclockwise direction along the circle, to arrive at the point $P = (x, y)$. In this sense, the length $s = t$ units is being **wrapped** around the unit circle.

If $t < 0$, we begin at the point $(1, 0)$ on the unit circle and travel $s = |t|$ units in the clockwise direction to arrive at the point $P = (x, y)$. See Figure 18(b).

Figure 18



If $t > 2\pi$ or if $t < -2\pi$, it will be necessary to travel around the unit circle more than once before arriving at the point P. Do you see why?

Let's describe this process another way. Picture a string of length $s = |t|$ units being wrapped around a circle of radius 1 unit. We start wrapping the string around the circle at the point $(1, 0)$. If $t \geq 0$, we wrap the string in the counterclockwise direction; if $t < 0$, we wrap the string in the clockwise direction. The point $P = (x, y)$ is the point where the string ends.

This discussion tells us that, for any real number t, we can locate a unique point $P = (x, y)$ on the unit circle. We call P **the point on the unit circle that corresponds to *t*.** This is the important idea here. No matter what real number t is chosen, there is a unique point P on the unit circle corresponding to it. We use the coordinates of the point $P = (x, y)$ on the unit circle corresponding to the real number t to define the **six trigonometric functions of *t*.**

Let t be a real number and let $P = (x, y)$ be the point on the unit circle that corresponds to t.

The **sine function** associates with t the y-coordinate of P and is denoted by

$$\sin t = y$$

The **cosine function** associates with t the x-coordinate of P and is denoted by

$$\cos t = x$$

If $x \neq 0$, the **tangent function** associates with t the ratio of the y-coordinate to the x-coordinate of P and is denoted by

$$\tan t = \frac{y}{x}$$

Tan t = sin / cos

If $y \neq 0$, the **cosecant function** is defined as

$$\csc t = \frac{1}{y}$$

Csc t = 1/sin

If $x \neq 0$, the **secant function** is defined as

$$\sec t = \frac{1}{x}$$

Sec t = 1/cos

If $y \neq 0$, the **cotangent function** is defined as

$$\cot t = \frac{x}{y}$$

Cot t = cos / sin

365

Quadrant II

Angles between $\pi/2$ and π

Cos= x is negative and
Sin = y is positive
Tan= y/-x → negative
Cot = -x/y → negative

Quadrant I

Angles between 0 and $\pi/2$

cos= x is positive and
sin= y is positive
Tan= y/x → positive
Cot = x/y → positive

Quadrant III

Tan= -y/-x → positive

Angles between π and $3\pi/2$

Cot = -x/-y → positive
Cos= x is negative and
Sin = y is negative

Quadrant IV

Tan= -y/x → negative

Angles between $3\pi/2$ and 2π

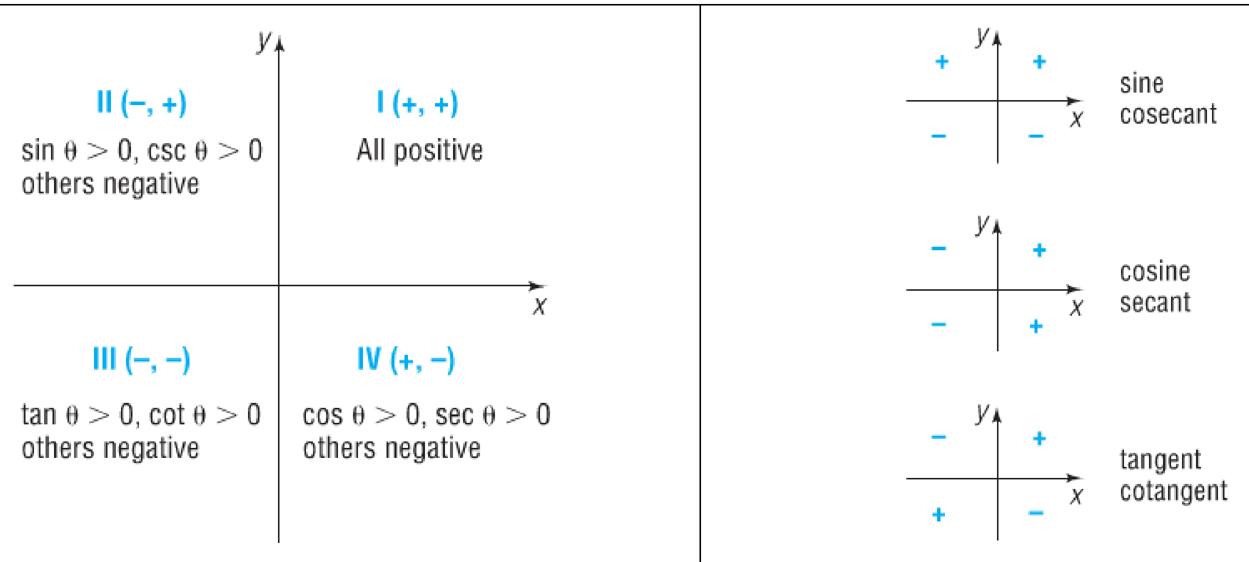
Cot = x/-y → negative
cos= x is positive and
sin= y is negative

Assign "x" to Cos and "y" to Sin

Cos and x are positive in Quadrant I, IV
Cos and x are negative in Quadrants II, III

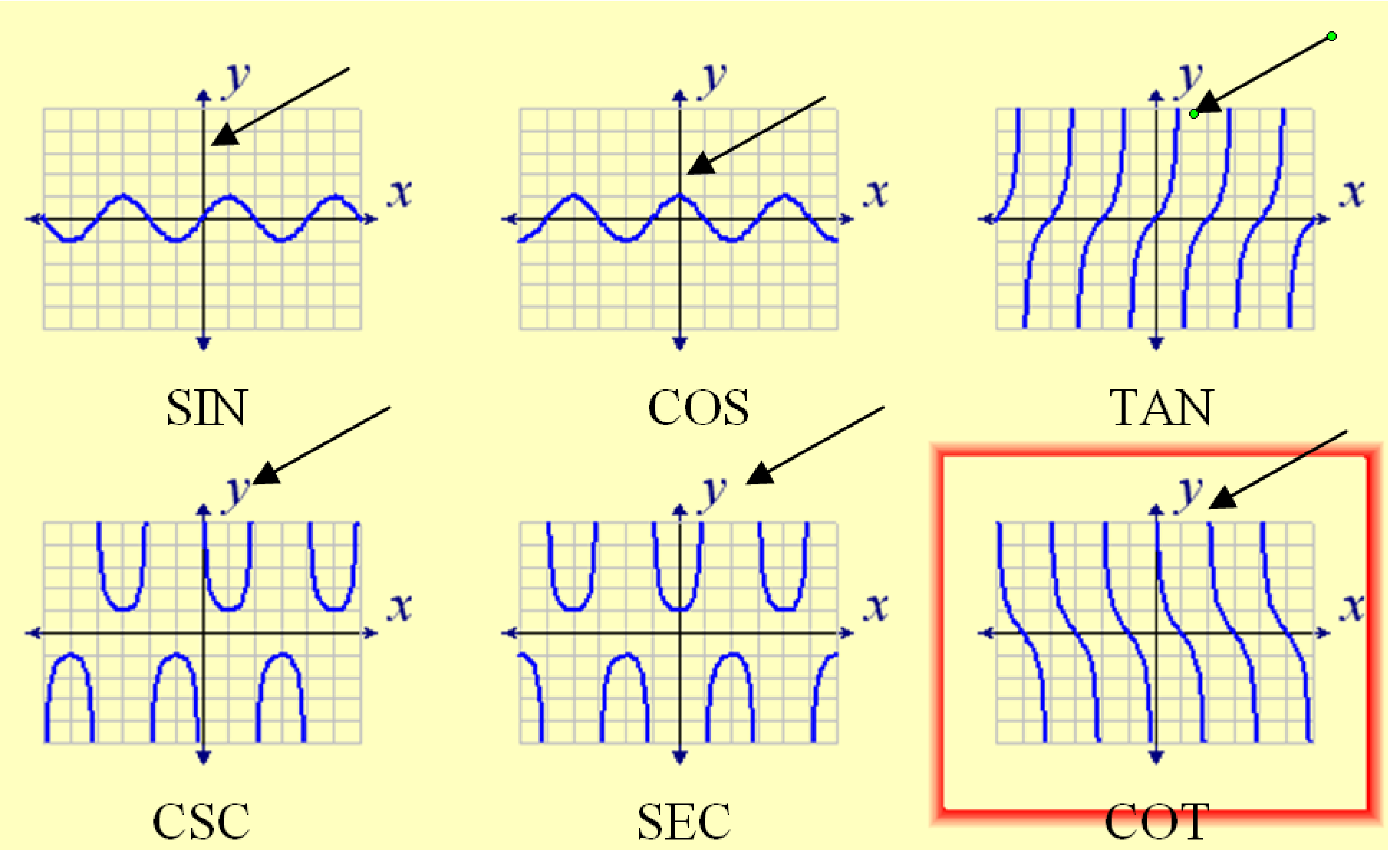
Sin and y are positive in Quadrants I, II
Sin and y are negative in Quadrants III, IV

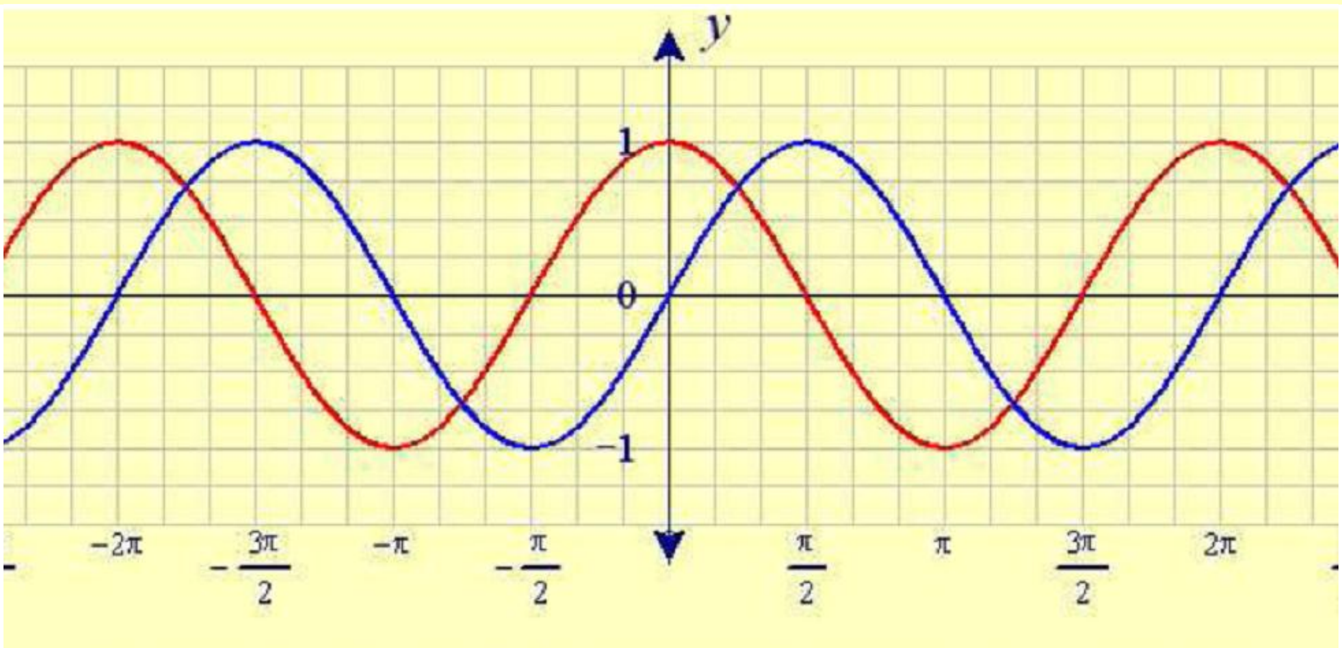
Quadrant of *P*	**sin θ, csc θ**	**cos θ, sec θ**	**tan θ, cot θ**
I	Positive	Positive	Positive
II	Positive	Negative	Negative
III	Negative	Negative	Positive
IV	Negative	Positive	Negative



(a) (b)

54. Name the quadrant in which the angle θ lies when $\cos\theta < 0$ and $\tan\theta < 0$.


y
x
SIN
COS
TAN
CSC
SEC
COT


y
1
0
−1
−2π
$-\frac{3\pi}{2}$
−π
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π

6.2 Trigonometric Functions of Acute Angles

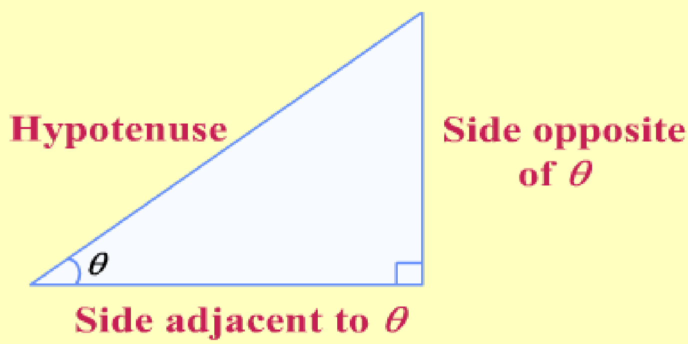


Figure 1: Legs Labeled Relative to θ

Sine, Cosine and Tangent

Assume θ is one of the acute (less than a right angle) angles in a right triangle, as in Figure 1, and let *adj* and *opp* stand for, respectively, the lengths of the sides adjacent and opposite of θ. Let *hyp* stand for the length of the hypotenuse of the right triangle. Then the **sine**, **cosine**, and **tangent** of θ, abbreviated sin θ, cos θ and tan θ, are the ratios:

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}},\ \cos(\theta) = \frac{\text{adj}}{\text{hyp}},\ \tan(\theta) = \frac{\text{opp}}{\text{adj}}.$$

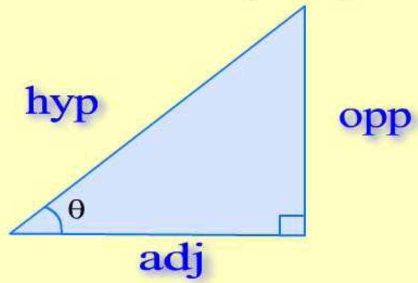
Cosecant, Secant, and Cotangent

Again, assume that θ is one of the acute angles in a right triangle, as in Figure 1. Then the **cosecant**, **secant** and **cotangent** of θ, abbreviated $\csc(\theta)$, $\sec(\theta)$, and $\cot(\theta)$, are the reciprocals, respectively, of $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.

That is,

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{\text{hyp}}{\text{opp}},\ \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{\text{hyp}}{\text{adj}},$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\text{adj}}{\text{opp}}.$$



$\sin\ \theta = \dfrac{\boxed{}}{\boxed{}}$ $\cos\ \theta = \dfrac{\boxed{}}{\boxed{}}$ $\tan\ \theta = \dfrac{\boxed{}}{\boxed{}}$

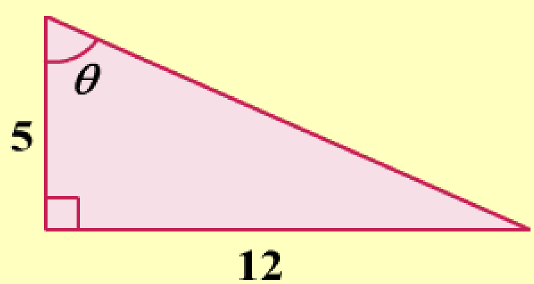
$\sec\ \theta$ $\sin\ \theta = \dfrac{1}{\boxed{}}$

$\cot\ \theta$ $\cos\ \theta = \dfrac{1}{\boxed{}}$

$\csc\ \theta$ $\tan\ \theta = \dfrac{1}{\boxed{}}$

Practice

Use the information contained in the figures to determine the values of six trigonometric functions of θ.

	$(\text{hyp})^2 = (\text{adj})^2 + (\text{opp})^2$
$\sin(\ \theta\ ) = \dfrac{\text{opp}}{\text{hyp}} =$ $\cos(\ \theta\ ) = \dfrac{\text{adj}}{\text{hyp}} =$ $\tan(\ \theta\ ) = \dfrac{\text{opp}}{\text{adj}} =$	S $\csc(\ \theta\ ) = \dfrac{1}{\sin(\ \theta\ )} =$ hyp / opp E $\sec(\ \theta\ ) = \dfrac{1}{\cos(\ \theta\ )} =$ hyp / adj O $\cot(\ \theta\ ) = \dfrac{1}{\tan(\ \theta\ )} =$ adj / opp

You were asked to find the arc-length of a circle with a radius of 31 ft and an angle of 14.

$$r = 31 \text{ ft}; \ \theta = 14$$

The arc-length of a circle can be found using the arc-length formula $s = r\,\theta$. In this case $r = 31$ ft and $\theta = 14$. Therefore,

$$s = (31)(14) = 434 \text{ ft}$$

Answer: 434 ft

6.2 Trigonometric Functions of Acute Angles

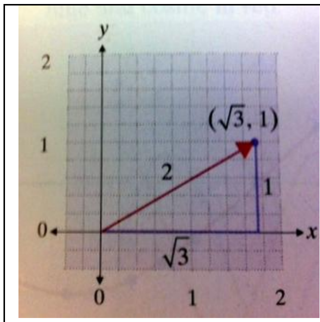
All problems need calculator so do not use.

6.3 Trigonometric Functions of Any Angle

In text

6.4 Graphs of Trigonometric Functions

6.5 in Sullivan



x	$\sin(x)$	$\cos(x)$	x	$\sin(x)$	$\cos(x)$
0	0	1	π	0	-1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\frac{\pi}{2}$	1	0	$\frac{3\pi}{2}$	-1	0
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{7\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

Figure 1: Selected Values of Sine and Cosine

θ (Radians)	θ (Degrees)	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\frac{\pi}{6}$	$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

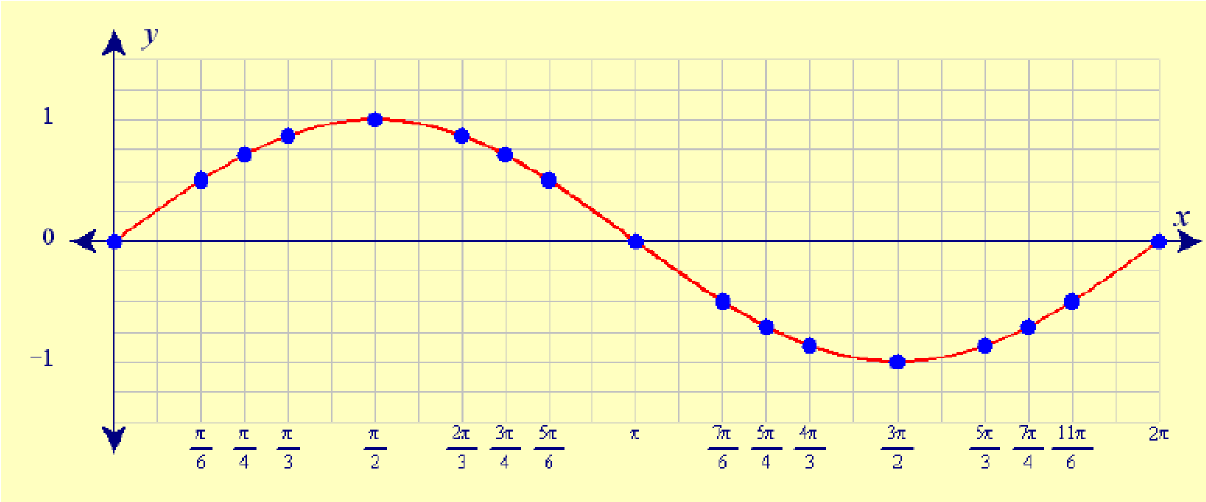


Figure 2: Graph of Sine between 0 and 2π

Y = sin x [the sin graph starts at the origin of the “y” axis]

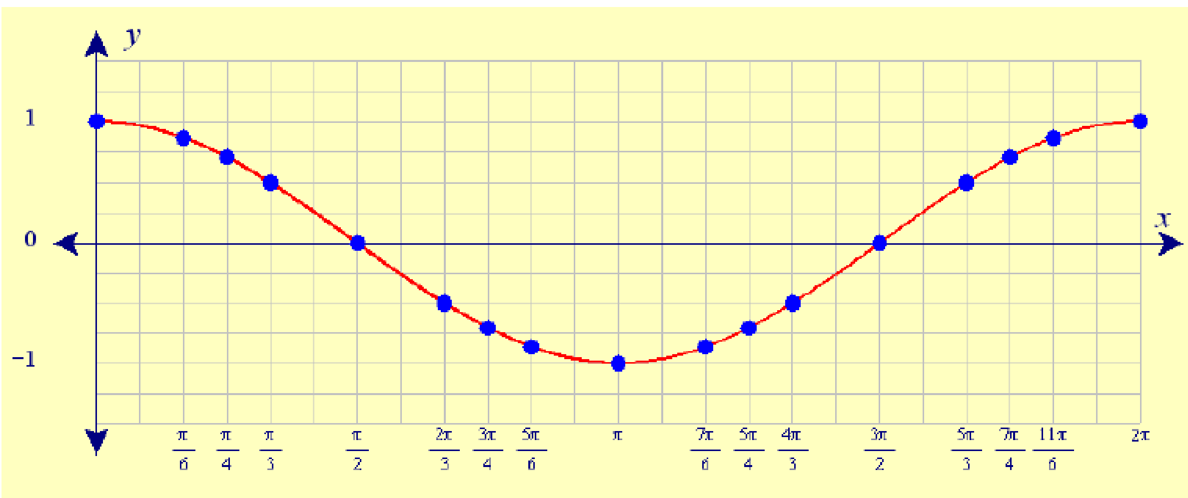


Figure 3: Graph of Cosine between 0 and 2π

Y= cos x [the cos graph starts at above the origin on the “y” axis]

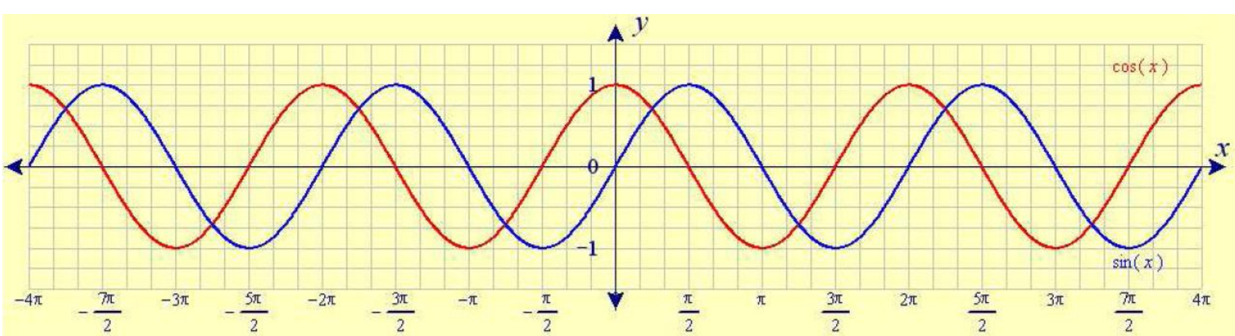
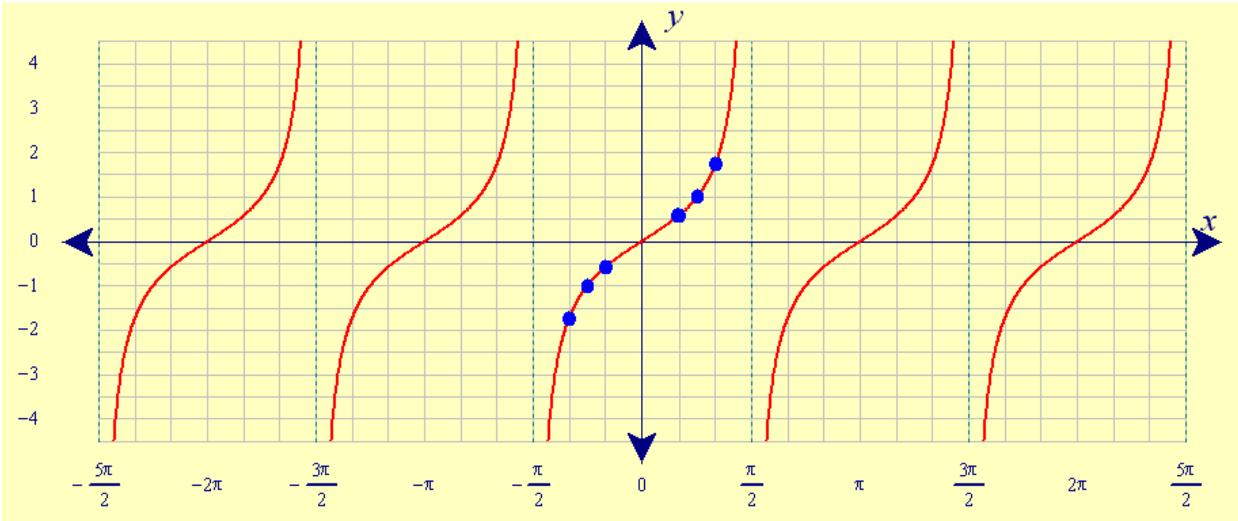


Figure 4: Graph of Cosine and Sine from -4π to 4π

Y = sin x and Y = cos x



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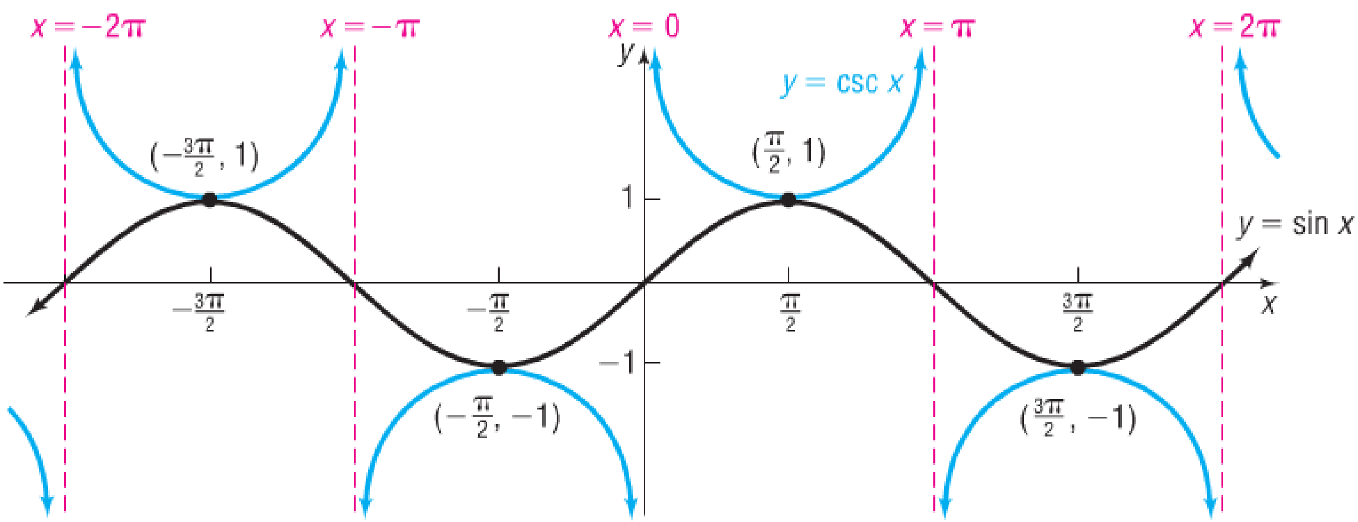
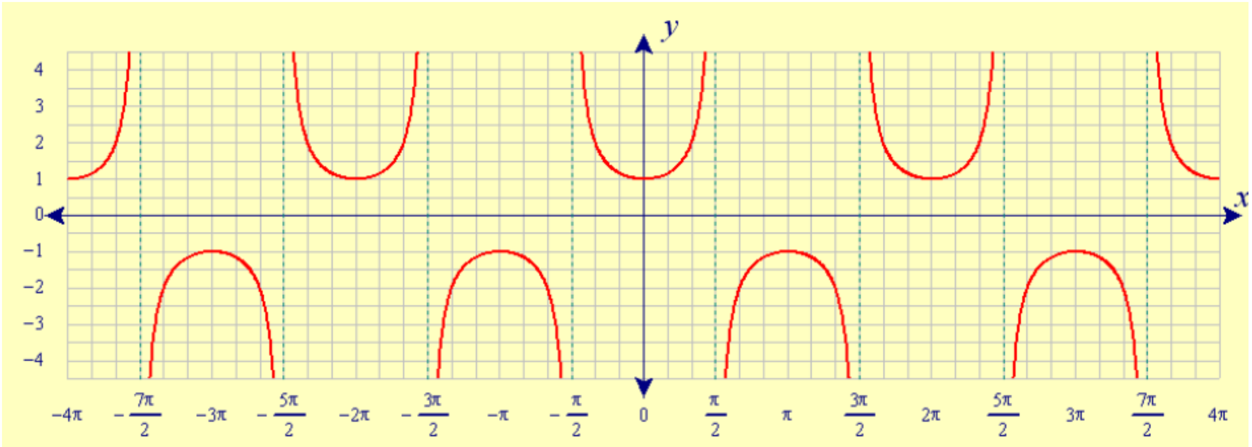
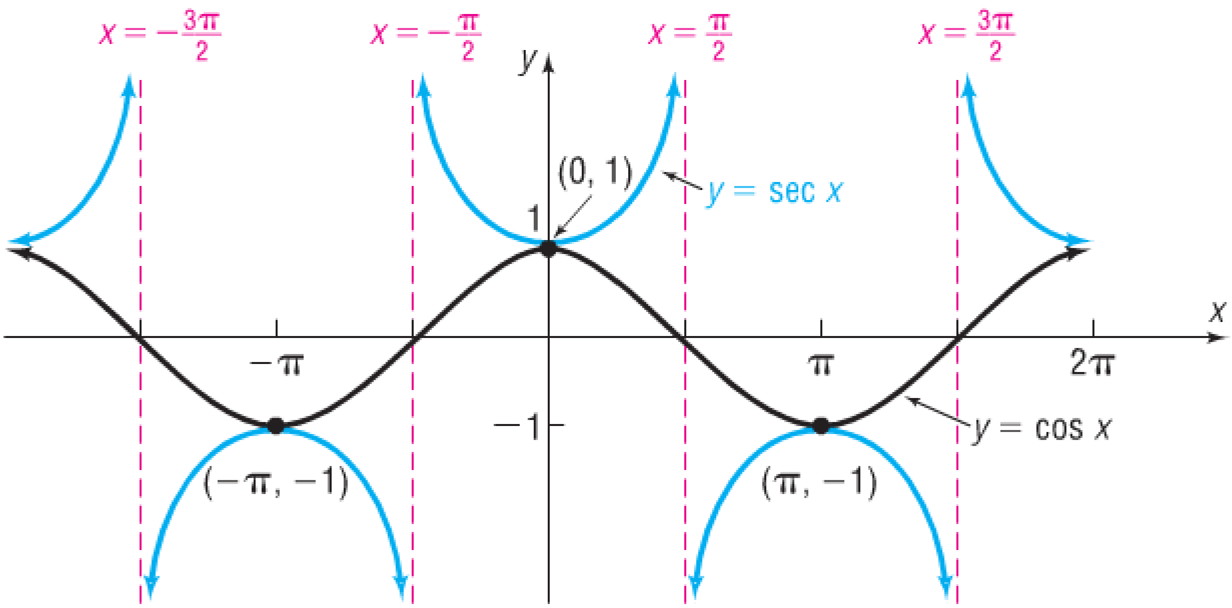
Y = tan x [similar to y=x^3 and it crosses the "y"axis]

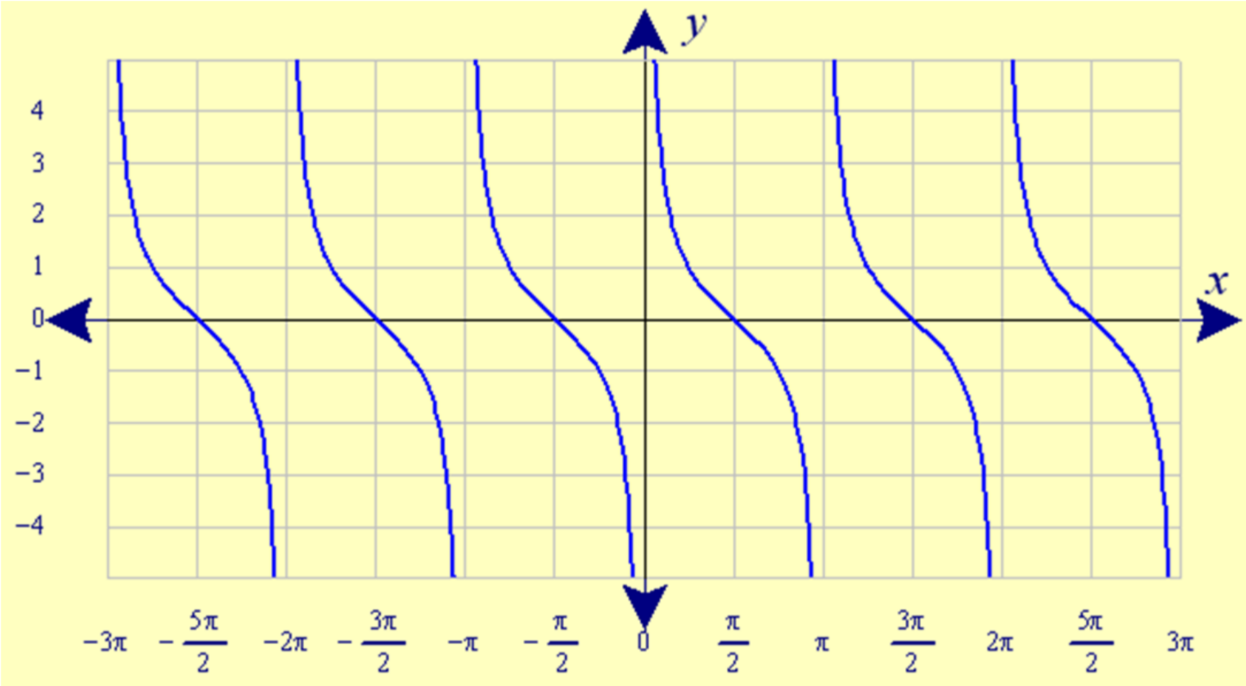


Y=csc x [complete parabola next to "y" axis]

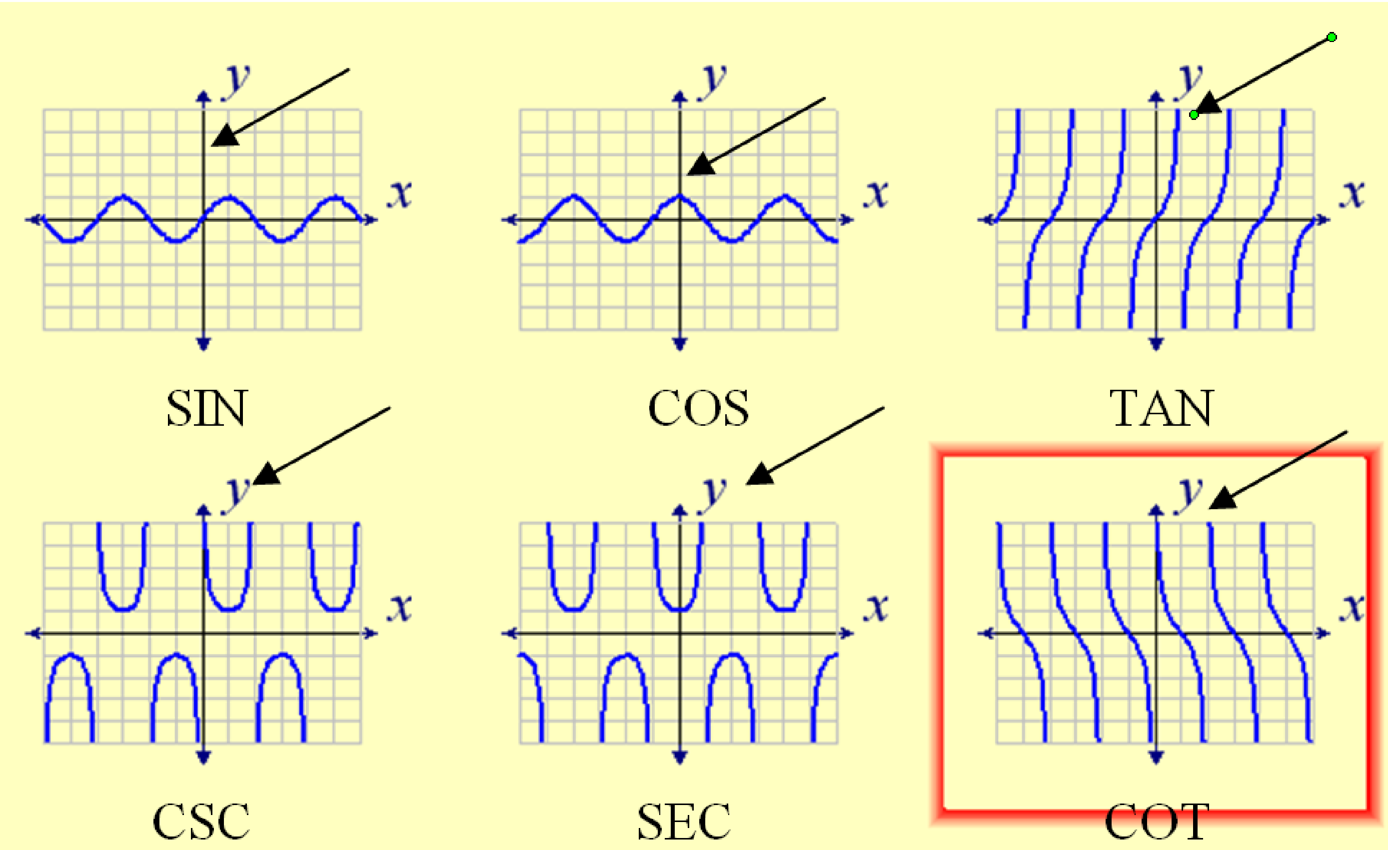




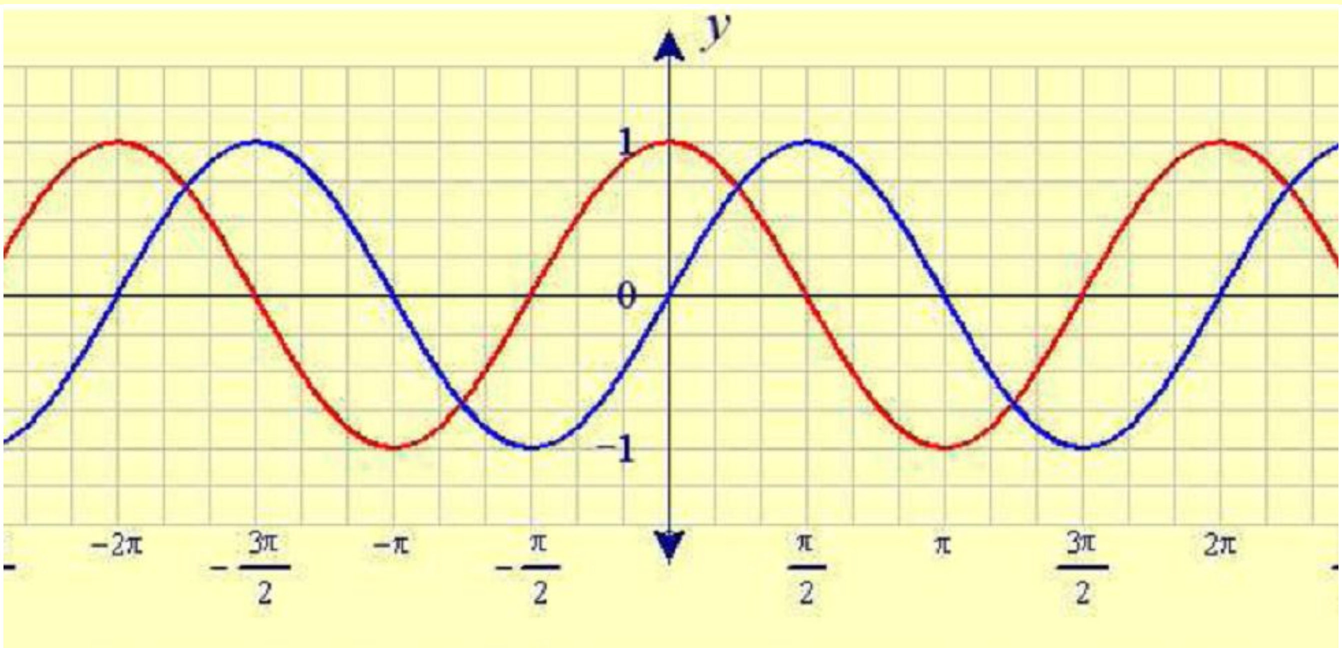
Y = sec x [a parabola which crosses the “y” axis]



Y= cot x [similar to the graph y= $-x^3$]



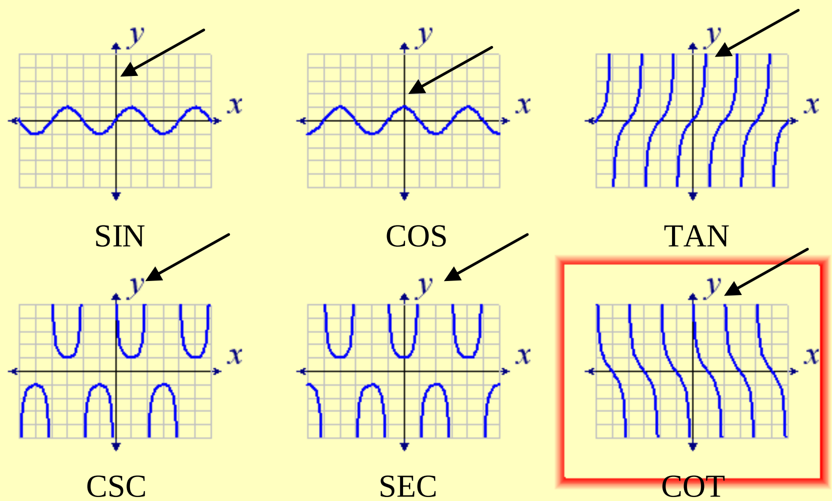
y
x
SIN
COS
TAN
CSC
SEC
COT



y
1
0
-1
−2π
−3π/2
−π
−π/2
π/2
π
3π/2
2π

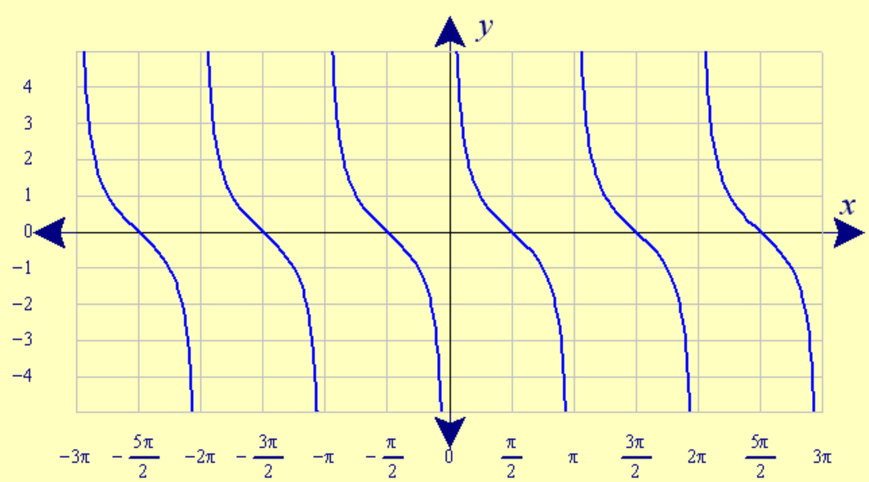
Graph the following function: $y = \frac{3}{2}\cot(x - \pi)$

Select the general shape of the graph of this function.



You were asked to graph the following function: $y = \frac{3}{2}\cot(x - \pi)$

The graph of $\cot(x)$ is



6.5 Inverse Trigonometric Functions

In text

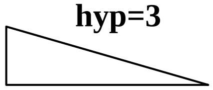
FINAL QUESTIONS

Quadrant of *P*	sin θ, csc θ	cos θ, sec θ	tan θ, cot θ
I	Positive	Positive	Positive
II	Positive	Negative	Negative
III	Negative	Negative	Positive
IV	Negative	Positive	Negative

56. If $\sin\theta = \frac{1}{3}$ and θ is in quadrant II, find all other trigonometric functions of θ.

In quadrant II sin is positive because sin = y and y is positive in quadrant II

Think of the problem as a triangle and we have the “hyp”, and “opp” due to info given in the problem. Find the “adj” side of the triangle using the Pythagorean theorem

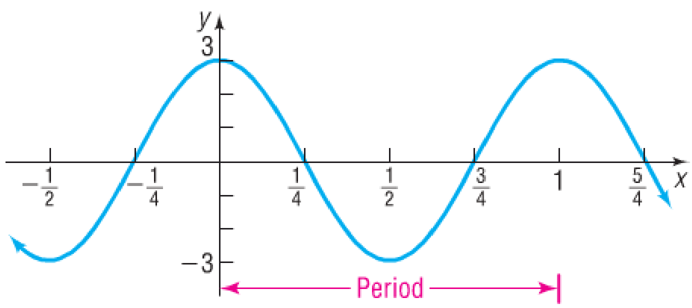
Think of the problem as a triangle	**(a) sinα =**	$\sin(\ \theta\ ) = \frac{\text{opp}}{\text{hyp}}$	**= 1/3**
hyp=3 **opp=1**  **adj= (Sqrt 8)**	**(b) cosα =**	$\cos(\ \theta\ ) = \frac{\text{adj}}{\text{hyp}}$	**= (Sqrt 8) / 3**
Pythagorean theorem	**(c) tanα =**	$\tan(\ \theta\ ) = \frac{\text{opp}}{\text{adj}}$	**= 1/ (Sqrt 8)**
a^2 + b^2 =c^2 opp^2 + adj^2 = hyp^2	**(d) cotα =**	$\csc(\ \theta\ ) = \frac{1}{\sin(\ \theta\ )} = \frac{\text{hyp}}{\text{opp}}$	= 3/1 = **3**
sin = 1/3 = opp/hyp 1^2+ adj^2 = 3^2	**(e) secα=**	$\sec(\ \theta\ ) = \frac{1}{\cos(\ \theta\ )} = \frac{\text{hyp}}{\text{adj}}$	**= 3 /(Sqrt 8)**
1 +adj^2 = 9 adj^2 = 9-1 adj^2 = 8 **adj = Sqrt 8**	**(f) cscα**	$\cot(\ \theta\ ) = \frac{1}{\tan(\ \theta\ )} = \frac{\text{adj}}{\text{opp}}$	= (Sqrt 8)/1 = **8**

57. Find the exact values of each of the remaining trigonometric functions of θ when $\tan\theta = -\frac{1}{8}$ and $\sec\theta < 0$.

QUADRANT II where tan is negative and sec is negative

Finding an Equation for a Sinusoidal Graph

Find an equation for the graph shown in Figure 60.



The graph has the characteristics of a cosine function. Do you see why? The maximum value, 3, occurs at $x = 0$. So we view the equation as a cosine function $y = A\cos(\omega x)$ with $A = 3$ and period $T = 1$. Then $\frac{2\pi}{\omega} = 1$, so $\omega = 2\pi$. The cosine function whose graph is given in Figure 60 is

$$y = A\cos(\omega x) = 3\cos(2\pi x)$$

THEOREM

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are given by

$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega} \tag{1}$$

Finding the Amplitude and Period of a Sinusoidal Function

Determine the amplitude and period of $y = 3\sin(4x)$.

Comparing $y = 3\sin(4x)$ to $y = A\sin(\omega x)$, we find that $A = 3$ and $\omega = 4$. From equation (1),

$$\text{Amplitude} = |A| = 3 \qquad \text{Period} = T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$$

If $\omega > 0$	$y = A\cos(\omega x)$	Amplitude $= \lvert A \rvert$ Period $= T = \dfrac{2\pi}{\omega}$

y= 6 sin

(π x) // then 6 = A // then $\pi = \omega$ thus $2\pi / \pi = 2$ thus 2 = T = Period

Amplitude is 6 and Period = 2

/////////////

Determine the amplitude and the period of the function without graphing.

(a) $y = -5\cos(6x)$

(b) $y = 3\sin(\pi x)$

(c) $y = \frac{1}{3}\sin(2x)$

(d) $y = \cos(\frac{x}{\pi})$

A) A = -5 T= 1 π /3 = π/3	C) A = 1/3 T= π
B) A = 3 T= 2	[D) A = 1 T=

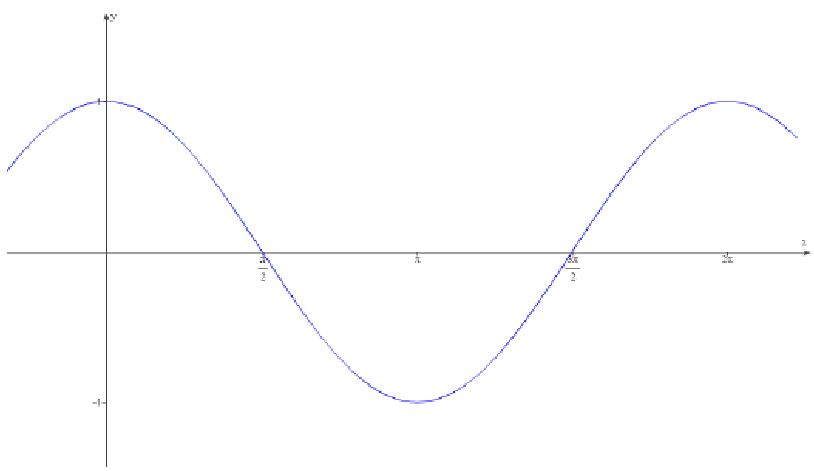
Which function matches the graph shown in the following graph ?

(a) $y = \cos x$

(b) $y = \cos 2x$

(c) $y = \sin 2x$

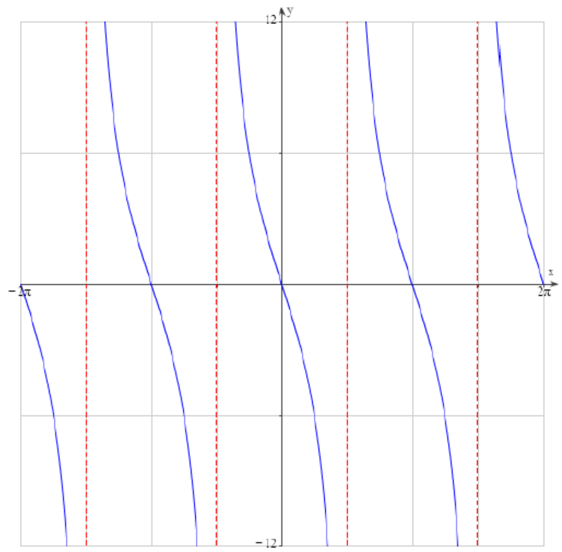
(d) $y = \sin x$



Y=cos2x

61. Which function matches the graph shown in the following graph ?

(a) $y = \tan x$ (b) $y = -\tan 6x$ (c) $y = -\cot x$ (d) $y = \cot 6x$

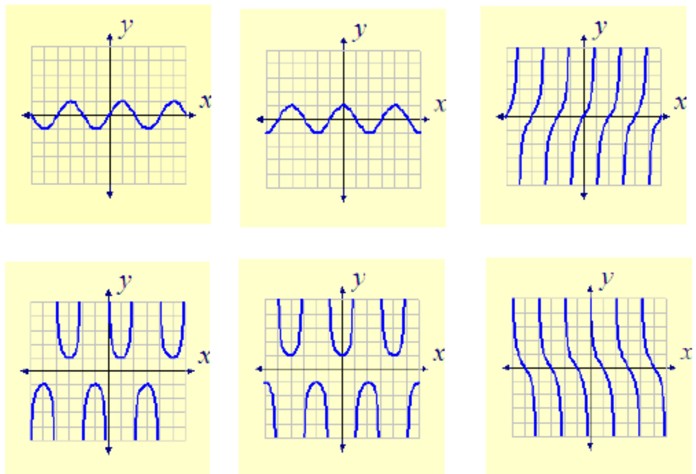


Y=-tan6x

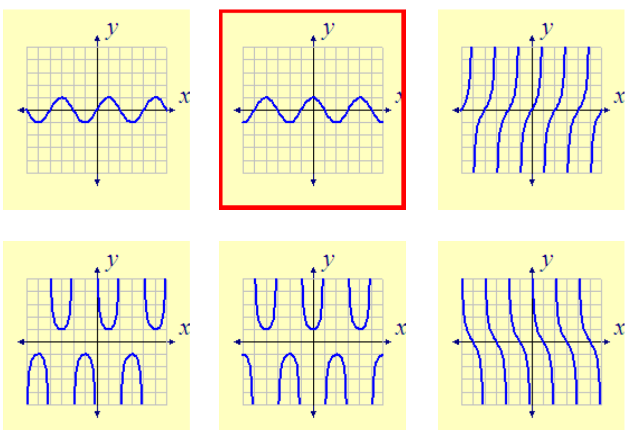
Graph the following function $y = 3\cos x + \frac{\pi}{3}$.

Step 1 .Select the general shape of the graph of this function.

Step 2. Determine how the general shape of the graph, chosen in the previous step, would be shifted, stretched, and reflected for the given function.



Step 1:



Step 2:

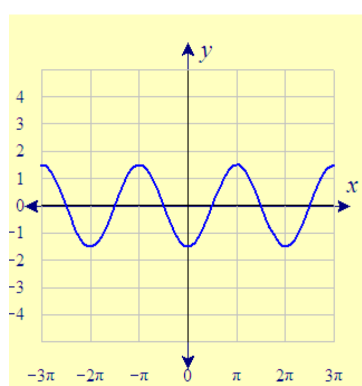
☐ Reflect graph across x-axis

Shift vertically: A) Up B) Down **C) None**

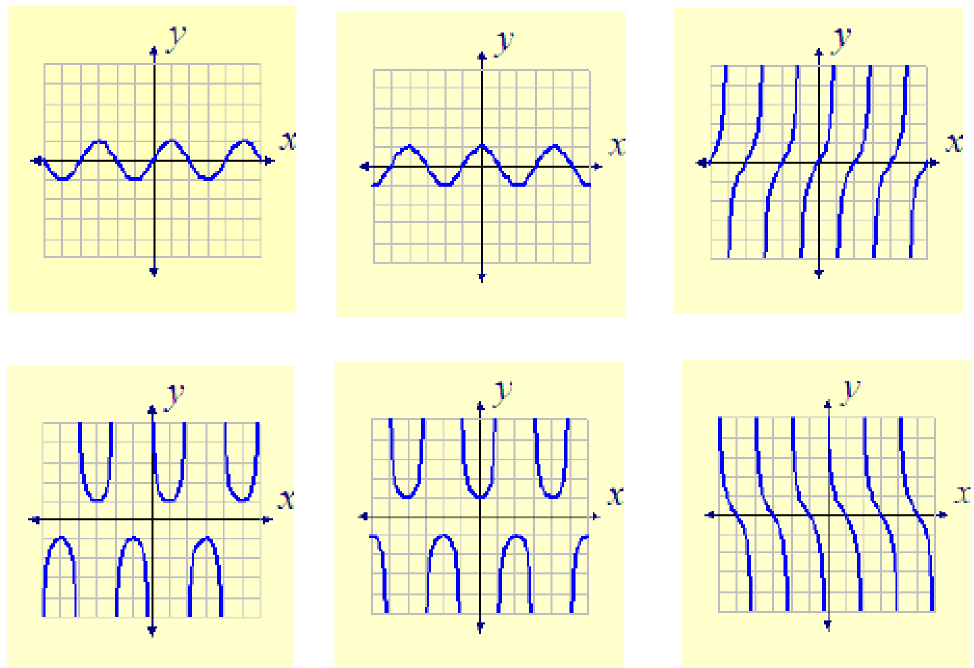
Shift horizontally: **A) Left** B) Right C) None $\boxed{\frac{\pi}{3}}$

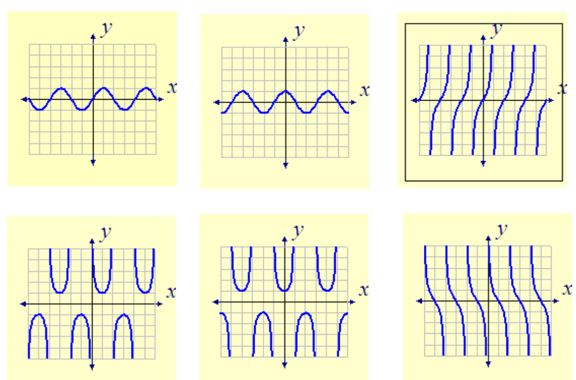
Stretch vertically: **A) Yes** B) No Factor: $\boxed{3}$

Stretch horizontally: A) Yes **B) No**



Select the general shape of the graph of the function $y = 2 + \tan x$.





67. Use trigonometric identities, to solve the following trigonometric equation on the interval $[0, 2\pi]$.

(a) $5\cos(-x) = 3\cos(x) + 1$

(b) $2\sin^2 x - 1 = 0$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Even–Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta \qquad \sec(-\theta) = \sec\theta \qquad \cot(-\theta) = -\cot\theta$$

(a) Simplify $\dfrac{\cot\theta}{\csc\theta}$ by rewriting each trigonometric function in terms of sine and cosine functions.

(b) Show that $\dfrac{\cos\theta}{1+\sin\theta} = \dfrac{1-\sin\theta}{\cos\theta}$ by multiplying the numerator and denominator by $1-\sin\theta$.

(c) Simplify $\dfrac{1+\sin u}{\sin u} + \dfrac{\cot u - \cos u}{\cos u}$ by rewriting the expression over a common denominator.

(d) Simplify $\dfrac{\sin^2 v - 1}{\tan v \sin v - \tan v}$ by factoring.

(a) $$\frac{\cot\theta}{\csc\theta} = \frac{\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}} = \frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{1} = \cos\theta$$

(b) $$\frac{\cos\theta}{1+\sin\theta} = \frac{\cos\theta}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta} = \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

Multiply by a well-chosen 1: $\dfrac{1-\sin\theta}{1-\sin\theta}$.

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta} = \frac{1-\sin\theta}{\cos\theta}$$

(c) $$\frac{1+\sin u}{\sin u} + \frac{\cot u - \cos u}{\cos u} = \frac{1+\sin u}{\sin u}\cdot\frac{\cos u}{\cos u} + \frac{\cot u - \cos u}{\cos u}\cdot\frac{\sin u}{\sin u}$$

$$= \frac{\cos u + \sin u\cos u + \cot u\sin u - \cos u\sin u}{\sin u\cos u} = \frac{\cos u + \dfrac{\cos u}{\sin u}\cdot\sin u}{\sin u\cos u}$$

$\cot u = \dfrac{\cos u}{\sin u}$

$$= \frac{\cos u + \cos u}{\sin u\cos u} = \frac{2\cos u}{\sin u\cos u} = \frac{2}{\sin u}$$

(d) $$\frac{\sin^2 v - 1}{\tan v\sin v - \tan v} = \frac{(\sin v + 1)(\sin v - 1)}{\tan v(\sin v - 1)} = \frac{\sin v + 1}{\tan v}$$

Use trigonometric identities to simplify the expression.

(a) $\sec x \cos x$ (b) $\frac{1}{\sec^2\theta - 1}$ (c) $\frac{\sec\theta}{\csc\theta}$ (d) $\csc(x+2\pi)\sin x$ (e) $\frac{\sin(\beta)\tan(\frac{\pi}{2}-\beta)}{\cos(\beta)}$	[A] sec(x)cos(x) $Answer: \frac{1}{\cos(x)} * \cos(x) = 1$
[B] $\frac{1}{\sec^2(Á) - 1}$ $\frac{1}{\tan^2\beta}$ *or* Answer:= $\cot^2(x)$	[C] $\frac{\sec(\theta)}{\csc(\theta)}$ $\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}$ = tanθ
[D] $\csc(x+2\pi)\sin(x)$ =csc(x)*sin(x) =1/sin(x) * sinx Answer:=1	[E] $\frac{\sin(\beta)\tan\left(\frac{\pi}{2}-\beta\right)}{\cos(\beta)}$ $= \frac{\sin\beta\cot\beta}{\cos\beta}$ $= \frac{\sin\beta(\cos\beta/\sin\beta)}{\cos\beta}$ $= \frac{Cos\beta}{\cos\beta}$ $Answer = 1$
(f) $\cos(\frac{5\pi}{6} - \frac{7\pi}{6})$	

sum and difference identities

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

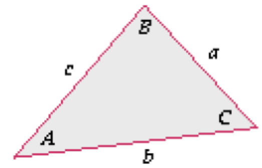
$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

the laws of sines and cosines

the law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



the law of cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

area of a triangle (sine formula)

$$\text{Area} = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$$

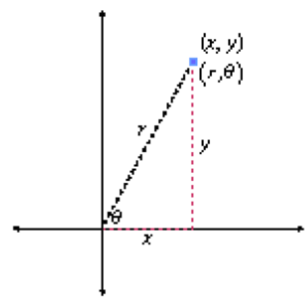
double-angle identities

$$\sin 2u = 2\sin u \cos u$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

$$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$$

polar coordinates



$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x} \quad (x \neq 0)$$

power-reducing identities

$$\sin^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

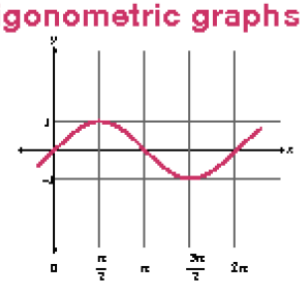
$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

half-angle identities

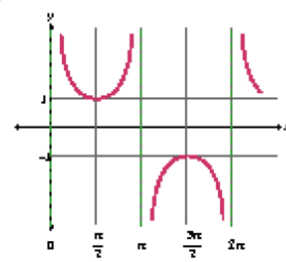
$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \qquad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

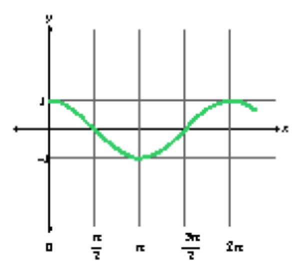
trigonometric graphs



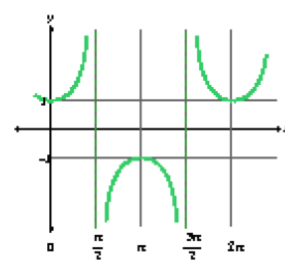
$y = \sin x$



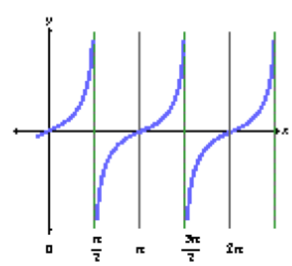
$y = \csc x$



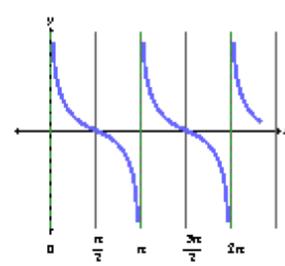
$y = \cos x$



$y = \sec x$



$y = \tan x$



$y = \cot x$

product-to-sum identities

$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\cos x \sin y = \frac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$$

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

sum-to-product identities

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

radian and degree measure

$180° = \pi$

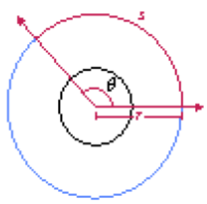
$1° = \frac{\pi}{180}$ $\qquad \left(\frac{180}{\pi}\right)^{\circ} = 1$

$x° = (x)\left(\frac{\pi}{180}\right)\text{ rad} \qquad x\text{ rad} = x\left(\frac{180}{\pi}\right)^{\circ}$

$s = \left(\frac{\theta}{2\pi}\right)(2\pi r)$

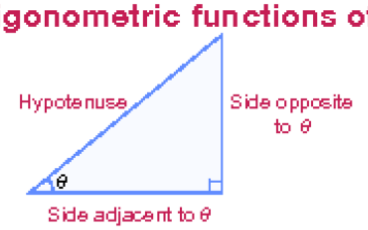
$= r\theta$

$A = \left(\frac{\theta}{2\pi}\right)(\pi r^2) = \frac{r^2\theta}{2}$



$\omega = \frac{\theta}{t} \qquad v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$

trigonometric functions of acute angles

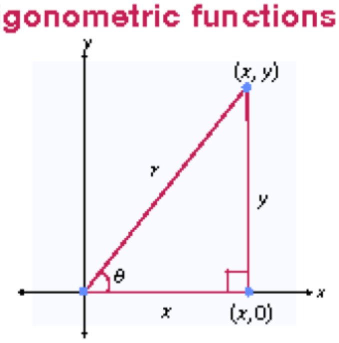


$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}}$

trigonometric functions of any angle



$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}\ (\text{for } y \neq 0)$

$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}\ (\text{for } x \neq 0)$

$\tan\theta = \frac{y}{x}\ (\text{for } x \neq 0) \qquad \cot\theta = \frac{x}{y}\ (\text{for } y \neq 0)$

cofunction identities

$\sin x = \cos\left(\frac{\pi}{2} - x\right) \qquad \csc x = \sec\left(\frac{\pi}{2} - x\right)$

$\cos x = \sin\left(\frac{\pi}{2} - x\right) \qquad \sec x = \csc\left(\frac{\pi}{2} - x\right)$

$\cot x = \tan\left(\frac{\pi}{2} - x\right) \qquad \tan x = \cot\left(\frac{\pi}{2} - x\right)$

reciprocal identities

$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$

quotient identities

$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$

period identities

$\sin(x + 2\pi) = \sin(x) \qquad \csc(x + 2\pi) = \csc(x)$

$\cos(x + 2\pi) = \cos(x) \qquad \sec(x + 2\pi) = \sec(x)$

$\tan(x + \pi) = \tan(x) \qquad \cot(x + \pi) = \cot(x)$

even/odd identities

$\sin(-x) = -\sin x \qquad \csc(-x) = -\csc x$

$\cos(-x) = \cos x \qquad \sec(-x) = \sec x$

$\tan(-x) = -\tan x \qquad \cot(-x) = -\cot x$

pythagorean identities

$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$

commonly encountered angles

θ	Radians	Sin θ	Cos θ	Tan θ
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	—
180°	π	0	–1	0
270°	$\frac{3\pi}{2}$	–1	0	—