Sin Graphs

Table 6

x	$y = \sin x$	(x, y)
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
π	0	$(\pi, 0)$
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	-1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
2π	0	$(2\pi, 0)$

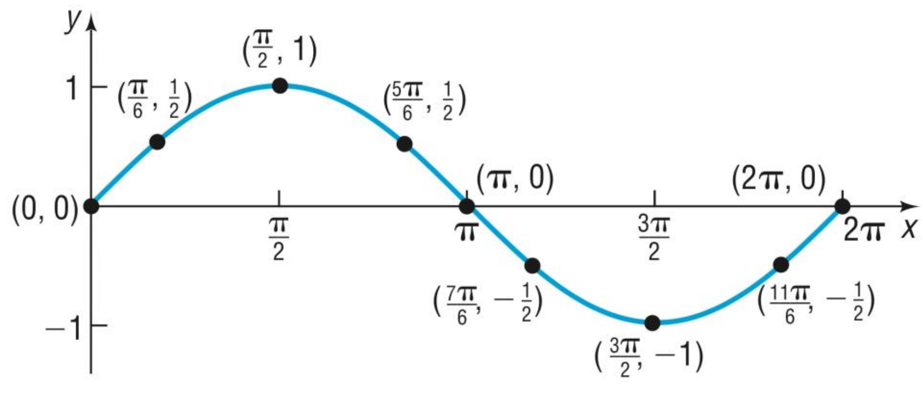


Figure 44 $y = \sin x, 0 \leq x \leq 2\pi$

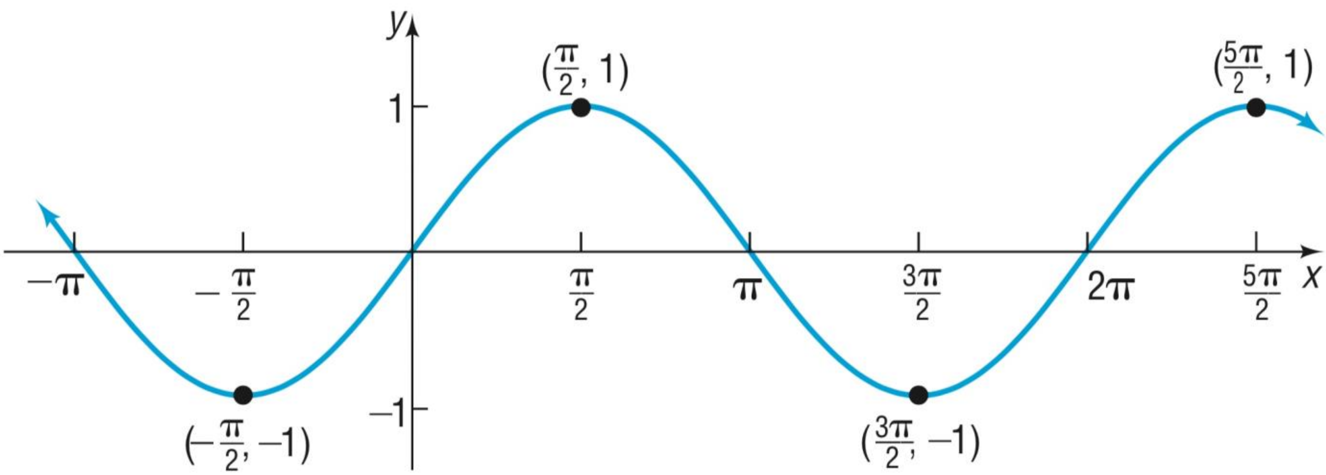


Figure 45 $y = \sin x, -\infty < x < \infty$

Properties of the Sine Function $y = \sin x$

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$; the y-intercept is 0.
6. The maximum value is 1 and occurs at $x = \ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$;

   the minimum value is -1 and occurs at $x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$.

Cos Graphs

Table 7

x	$y = \cos x$	(x, y)
0	1	$(0, 1)$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\left(\frac{\pi}{3}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2}, 0\right)$
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$
π	-1	$(\pi, -1)$
$\frac{4\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{4\pi}{3}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	0	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{5\pi}{3}$	$\frac{1}{2}$	$\left(\frac{5\pi}{3}, \frac{1}{2}\right)$
2π	1	$(2\pi, 1)$

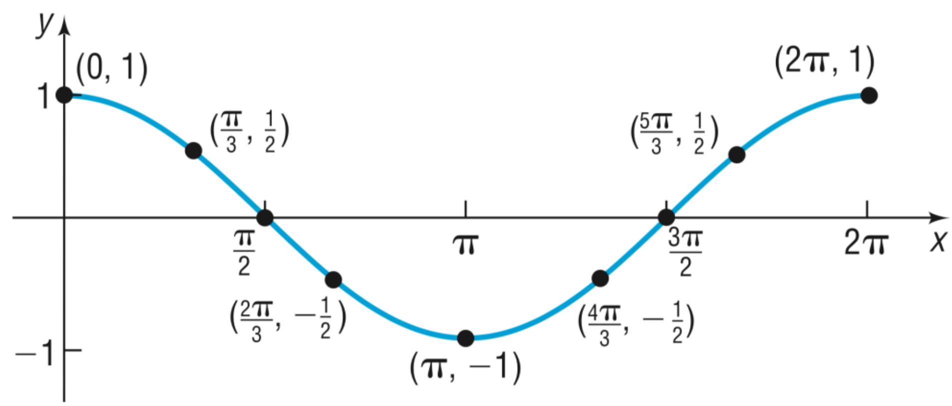


Figure 48 $y = \cos x, 0 \leq x \leq 2\pi$

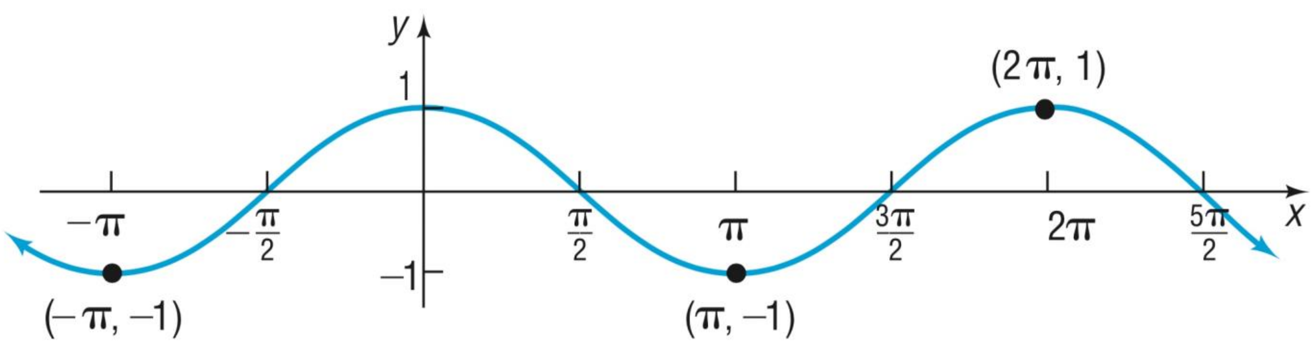


Figure 49 $y = \cos x, -\infty < x < \infty$

Properties of the Cosine Function

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the y-axis indicates.
4. The cosine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; the y-intercept is 1.
6. The maximum value is 1 and occurs at $x = \ldots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$; the minimum value is -1 and occurs at $x = \ldots, -\pi, \pi, 3\pi, 5\pi, \ldots$.

Amplitude and Period and Cycle

Sin

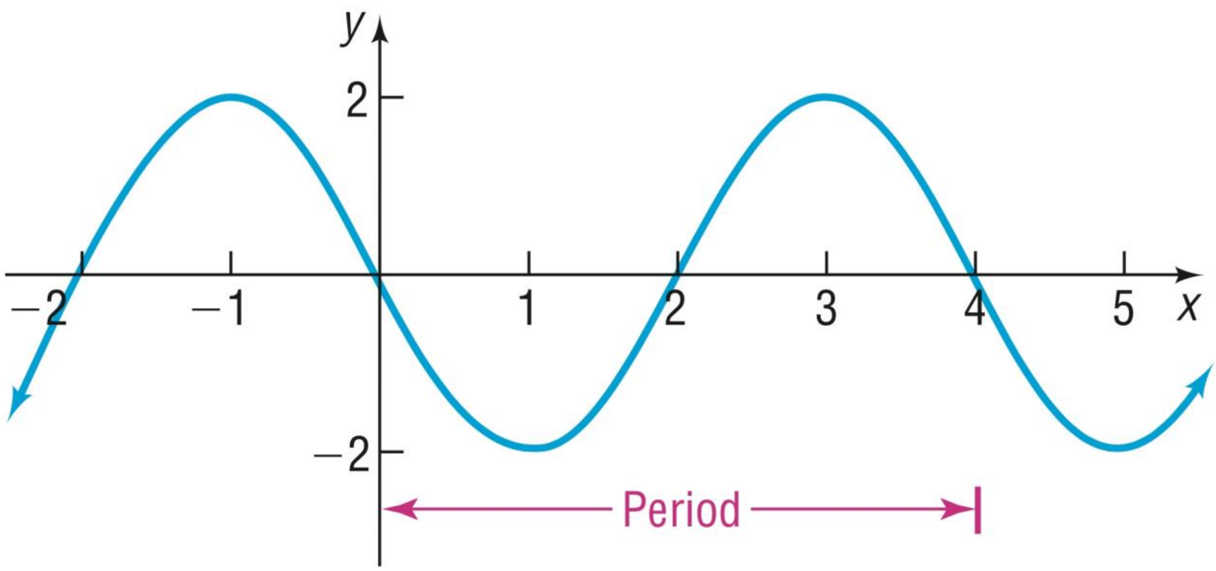


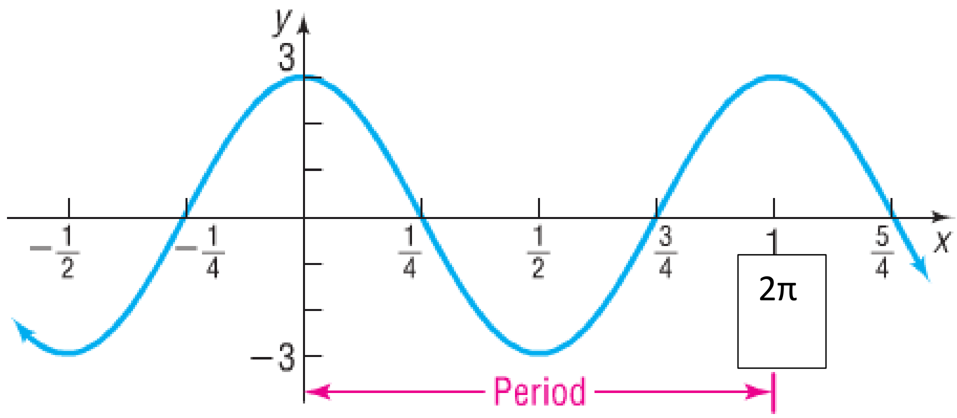
Figure 61

The graph is sinusoidal, with amplitude $|A| = 2$. The period is 4, so $\frac{2\pi}{\omega} = 4$, or $\omega = \frac{\pi}{2}$. Since the graph passes through the origin, it is easier to view the equation as a sine function,[†] but note that the graph is actually the reflection of a sine function about the x-axis (since the graph is decreasing near the origin). This requires that $A = -2$. The sine function whose graph is given in Figure 61 is

$$y = A\sin(\omega x) = -2\sin\left(\frac{\pi}{2}x\right)$$

T=4 then 4/1 = 2pi /w then cross multiply and solve for w. Answer us pi /2

Cos



The graph has the characteristics of a cosine function. Do you see why? The maximum value, 3, occurs at $x = 0$. So we view the equation as a cosine function $y = A\cos(\omega x)$ with $A = 3$ and period $T = 1$. Then $\frac{2\pi}{\omega} = 1$, so $\omega = 2\pi$. The cosine function whose graph is given in Figure 60 is

$$y = A\cos(\omega x) = 3\cos(2\pi x)$$

T=1 then 1/1 = 2pi /w then cross multiply and solve for w. Answer us 2pi

THEOREM

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are given by

$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega} \qquad \textbf{(1)}$$

Determine the amplitude and period of $y = 3\sin(4x)$.

Comparing $y = 3\sin(4x)$ to $y = A\sin(\omega x)$, we find that $A = 3$ and $\omega = 4$. From equation (1),

$$\text{Amplitude} = |A| = 3 \qquad \text{Period} = T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$$

If $\omega > 0$	$y = A\cos(\omega x)$	Amplitude $= \lvert A \rvert$ Period $= T = \frac{2\pi}{\omega}$

Y = 6 sin (πx)

// then 6 = A // then π = ω thus 2 π / π = 2 thus 2 = T = Period

Amplitude is 6 and Period = 2

Exercises

Determine the amplitude and the period of the function without graphing.

(a) $y = -5\cos(6x)$ (b) $y = 3\sin(\pi x)$ (c) $y = \frac{1}{3}\sin(2x)$ (d) $y = \cos(\frac{x}{\pi})$

Which function matches the graph shown in the following graph ?

(a) $y = \cos x$ (b) $y = \cos 2x$ (c) $y = \sin 2x$ (d) $y = \sin x$