CUNY YORK COLLEGE — FINAL EXAM REVIEW

MATH 120: *Precalculus*

Use the following questions to review for your final examination for Math 120. Your ability to answer these questions will reflect what you learned and understood in the course. The answer key included at the end of this packet includes most of the answers. Feel free to visit the Math Learning Center (MLC - 3E07C) to get tutoring if it's needed. Inquire with them what times are alloted for tutoring precalculus students. Send any comments or concerns about this review sheet to vthompson@york.cuny.edu. I hope you do well on the final:)

1. OBJECTIVE: Linear Equations

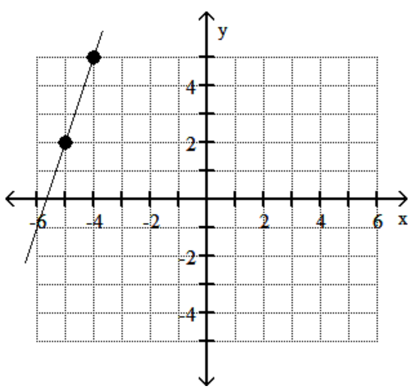
Find an equation for the line with the given properties from questions 1 - 8.

1) Slope undefined; containing the point (-3, -5)

2) Containing the points (-4, 0) and (5, 8);

3) Horizontal; containing the point (2, -6)

4)



5) Vertical line; containing the point (6, -4)

6) Slope = -8; y-intercept = 16

7) Parallel to the line $-5x - 6y = 6$; x-intercept = -6

8) Perpendicular to the line $y = 3x - 4$; containing the point (2, 4).
Find also the intercepts of both equation. Graph both lines on the same coordinate axis.

Solve the problem.

9) Consider the following equation of a line that is in the general form: $8x + 4y = 16$

a) Rewrite this equation in slope-intercept form.

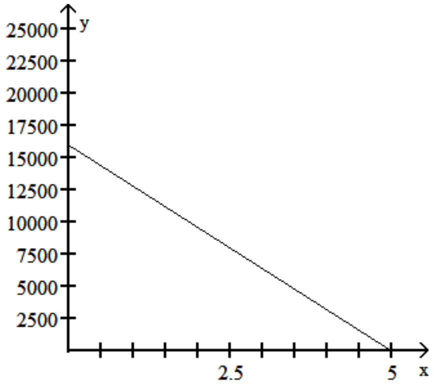
b) Find the equation in slope-intercept form for the line which is parallel to this line and passes through the point (-7,8)

Decide whether the pair of lines is parallel, perpendicular, or neither.

10) $3x - 6y = 5$
$18x + 9y = 11$

Solve the problem.

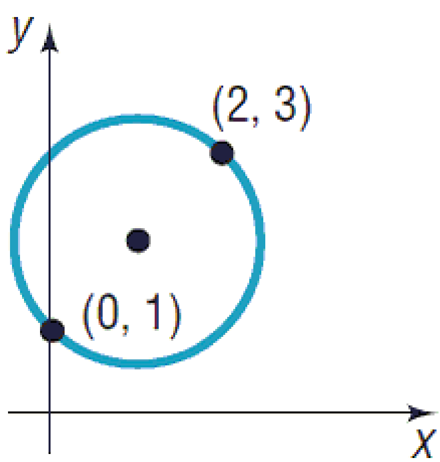
11) A school has just purchased new computer equipment for $16,000.00. The graph shows the depreciation of the equipment over 5 years. The point (0, 16,000) represents the purchase price and the point (5, 0) represents when the equipment will be replaced. Write a linear equation in slope-intercept form that relates the value of the equipment, y, to years after purchase x . Use the equation to predict the value of the equipment after 4 years.



2. OBJECTIVE: *Circles*

Find the center and radius of the given circle. Write the standard form of the equation of the circle.

12)



Write the standard form of the equation of the circle with radius r and center (h, k).

13) $r = \sqrt{14}$; $(h, k) = (4, 6)$

Solve the problem.

14) Find the equation of a circle in standard form where C(6, -2) and D(-4, 4) are endpoints of a diameter.

Find the center (h, k) and radius r of the circle. Find the intercepts. Graph the circle.

15) $x^2 + y^2 + 8x + 8y - 4 = 0$

3. OBJECTIVES: *Functions and their Graphs*

Determine whether the equation defines y as a function of x.

16) $y^2 + x = 8$

Determine whether the equation defines y as a function of x.

17) $y = -x^2 + 4x - 8$

Does the following have a function type relationship?

18) {(-2, 7), (1, 3), (3, -3), (8, -1)}

Determine whether the equation defines y as a function of x.

19) $y = \pm\sqrt{1 - 6x}$

Find the domain of the function.

20) $f(x) = \sqrt{13 - x}$

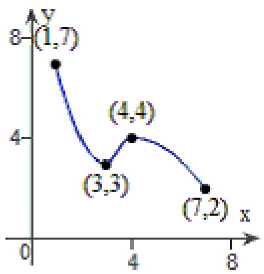
21) $g(x) = \dfrac{3x}{x^2 - 25}$

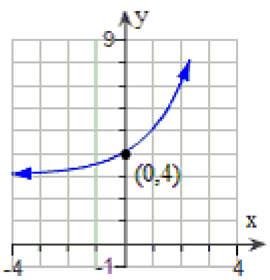
22) $f(x) = 7x - 4$

23) $\dfrac{x}{\sqrt{x - 6}}$

Using transformations, sketch the graph of the requested function.

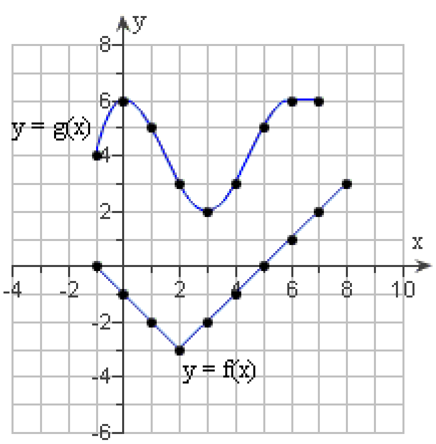
24) State the domain and range of the given graph of a function.

a) 

b) 

Evaluate the expression using the values given the graphs of f(x) and g(x).

25)



a) (f + g) (2) b) g(6) - f(0) c) (f/g) (7) d) f(-1) ∗ g(4)

e) (f o g) (6) f) (g o f) (8) g (f o f) (-1) h) (g o g) (0)

Evaluate the expression using the values given in the table.

26)

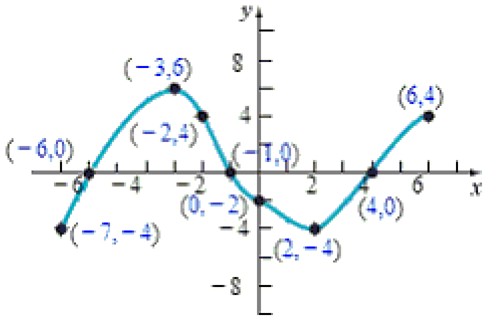
x	1	4	11	12
f(x)	-4	11	3	13

x	-5	-4	1	4
g(x)	1	-8	4	11

a) (f o g) (4) b) (fg) (4) c) (f-g)(4)

The graph of a function f is given. Use the graph to answer the question.

27)



a) Find f(2)

b) Is f(-4) positive or negative?

c) State the x and y intercepts.

d) State the domain and range of f(x).

e) Where is f(x) positive?

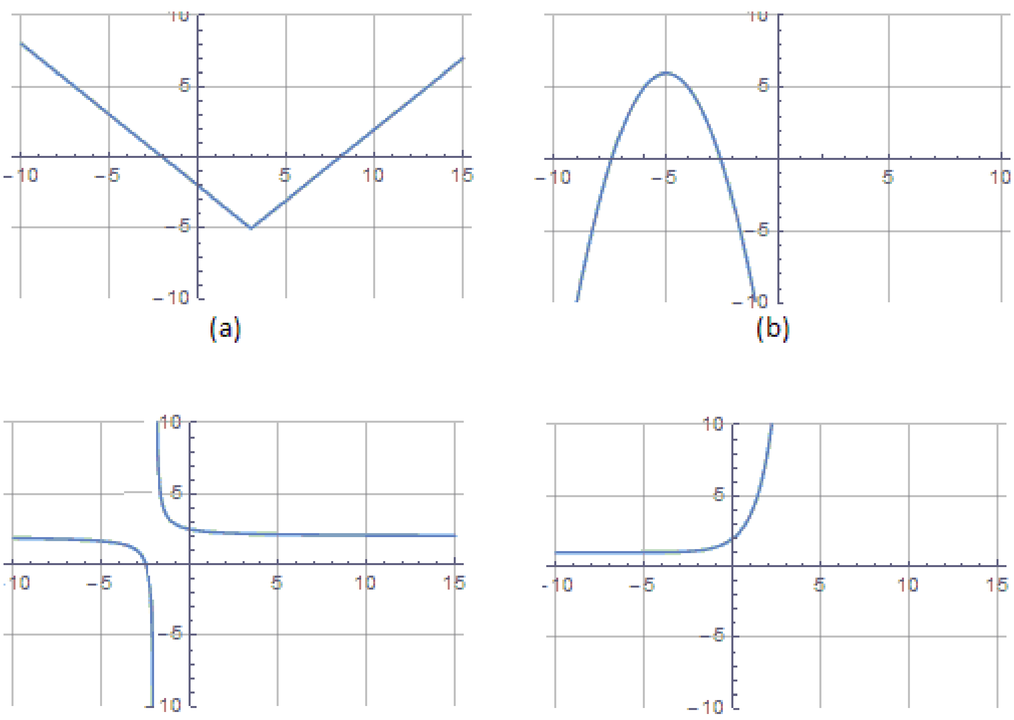
f) Where is f(x) negative?

g) State the intervals where f(x) is increasing, decreasing or is constant.

h) State the local minimum and local maximum values.

Match the graphs of (a) - (d) with the correct function listed in the box.

28)


(a) (b) (c) (d)

$y = e^x + 1$	$y = (x-3)^5 + 5$	$y = (x-5)^2 + 6$
$y = -(x+5)^2 + 6$	$y = \frac{1}{x+2}$	$y = e^x + 2$
$y = \lvert x+3 \rvert - 5$	$y = \lvert x-3 \rvert - 5$	$y = \frac{1}{x+2} + 2$

For the given functions f and g, find the requested composite function.

29) $f(x) = \frac{x - 7}{6}$, $g(x) = 6x + 7$; Find $(g \circ f)(x)$.

Answer the question about the given function.

30) Given the function $f(x) = 2x^2 - 6x - 3$, if $f(x) = -3$, what is x?

Determine algebraically whether the function is even, odd, or neither.

31) $f(x) = \frac{x}{x^2 + 4}$

32) $f(x) = 5x^3 - 2$

Solve the problem.

33) Along with incomes, people's charitable contributions have steadily increased over the past few years. The table below shows the average deduction for charitable contributions reported on individual income tax returns for the period 1993 to 1998. Find the average rate of change between 1995 and 1997.

Year	Charitable Contributions
1993	$1980
1994	$2350
1995	$2490
1996	$2760
1997	$3060
1998	$3120

Evaluate and graph the piecewise function.

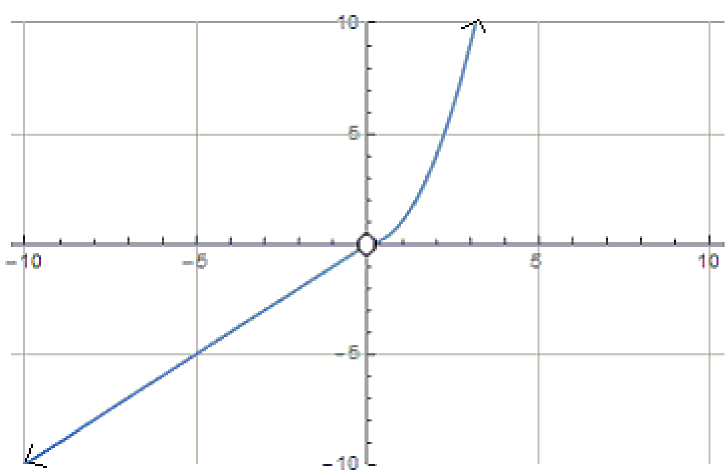
34)

$$f(x) = \begin{cases} -x + 2 & x < 0 \\ \sqrt{x} + 3 & x \geq 0 \end{cases}$$

Evaluate: a) f(0) b) f(4) c) f(-5)

Provide a possible piecewise function that will fit this graph.

35)



Find and simplify the difference quotient of f, $\frac{f(x + h) - f(x)}{h}$, $h \neq 0$, for the function.

36) $f(x) = x^2 + 5x + 6$

37) $f(x) = 8x - 1$

4. OBJECTIVE: *Quadratic Functions*

Solve the problem.

38) An object is thrown upward from an initial height of 160 feet with an initial velocity of 48 feet per second. The height h of the object after t seconds is given by the quadratic equation $h = -16t^2 + 48t + 160$.

a) What was the average rate of change from 3 to 4 seconds?

b) What was the maximum height of the object?

Answer the following:

39) $f(x) = x^2 - 8x + 7$

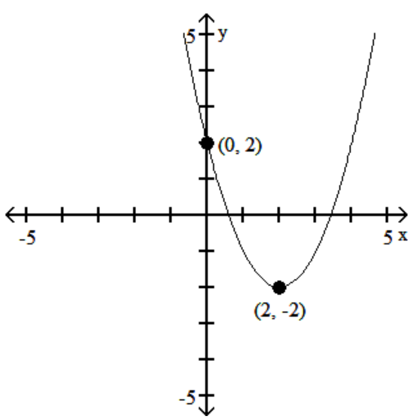
State the vertex ; axis of symmetry, intercepts and provide the graph of f(x) .

Answer the following:

40) Determine the equation of the quadratic function whose vertex is (-1,4) and crosses the x-axis at -3 and 1.

Determine the quadratic function whose graph is given.

41)



5. OBJECTIVES: *Polynomial and Rational Functions*

Use the Factor Theorem to determine whether x - c is a factor of f(x).

42) $f(x) = x^3 + 7x^2 - 16x + 18$; $x + 9$

List the potential rational zeros of the polynomial function. Do not find the zeros.

43) $f(x) = 6x^4 + 3x^3 - 4x^2 + 2$

Use the Rational Zeros Theorem to find all the real zeros of the polynomial function.

44) $f(x) = x^3 + 3x^2 - 4x - 12$

Given the function below, answer the following questions.

45) Let f(x) = $f(x) = x^2 (x-4)(x+1)$

a) Determine the end behavior of the f(x)?

b) At most how many turning points are there?

c) What are the intercepts?

d) Determine if the function crosses or touches the x-axis at the x-intercepts.

e) Determine the behavior of the graph near each of the the intercepts.

g) Provide the graph of f(x)

Find the asymptote, if any, of the function.

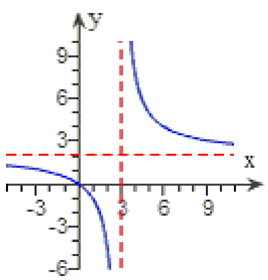
46) $h(x) = \dfrac{x^2 - 4}{(x+2)(x+3)}$

47) $g(x) = \dfrac{x^2 + 2x - 7}{x - 7}$

Use the graph shown to answer the following

48) (a) State the function that matches the graph

(b) State any asymtotes.



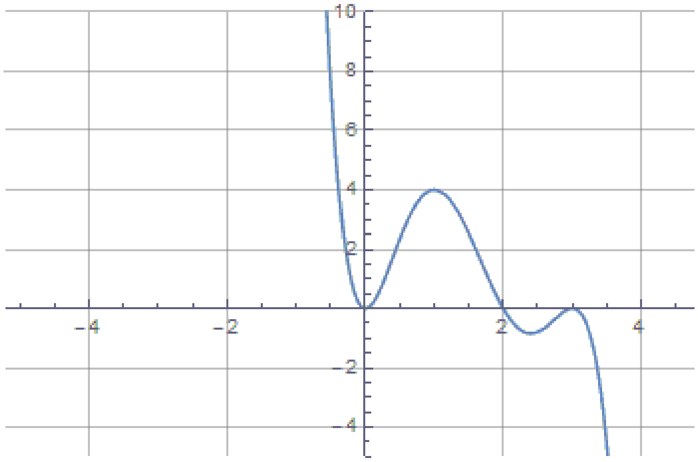
Answer the following

49) a) Construct a polynomial function f(x) that is of degree 3 with the zeros -3 (multiplicity 2), 2 (multiplicity 1) and f(x) goes to ∞ as x-> $-\infty$.

b) Construct a polynomial function f(x) that is of degree 2 with the zeros -4 and 3 f(x) goes to ∞ as x-> $-\infty$

Which of the following polynomial functions might have the graph illustrated below?

50)



a) $x^2(x-2)(x-3)$ b) $-x(x-2)^2(x-3)^2$ c) $-x^2(x-2)(x-3)^2$ d) $x(x-2)^2(x+3)$

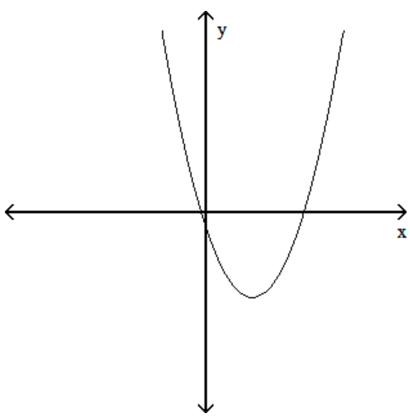
6. OBJECTIVES: The Inverse Function and Exponential Functions

Indicate whether the function is one-to-one.

51) {(6, -2), (5, -1), (3, 0), (1, 1)}

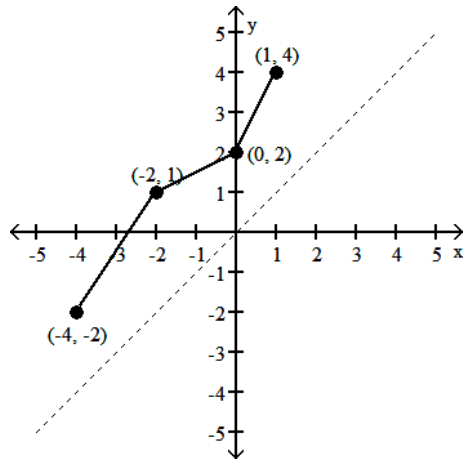
Use the horizontal line test to determine whether the function is one-to-one.

52)



Use the graph of the given one-to-one function to sketch the graph of the inverse function. For convenience, the graph of y = x is also given.

53)



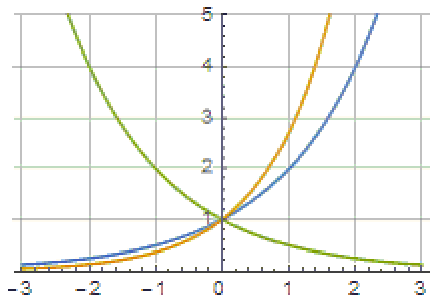
The function f is one-to-one. Find its inverse.

54) $f(x) = x^3 + 5$

55) $f(x) = \dfrac{5}{3x + 7}$

Exponential functions.

56) Three exponential functions are graphed below. They are $f(x)=e^x$, $g(x)=2^x$ and $h(x)=.5^x$. Which one is f(x), g(x) and h(x)? Label the graphs .



The graph of all exponential functions $f(x)=a^x$ (where $a > 0$; $a\neq1$) are common in comparison to each other . What are three different characteristics the graph of exponential functions share.? For example: All of them have are 1-1 functions.

Write as the sum and/or difference of logarithms. Express powers as factors.

57) $\log_w \left(\dfrac{11x}{5}\right)$

58) $\ln \sqrt[3]{ey}$

Express as a single logarithm.

59) $3 \log_6 x + 5 \log_6 (x - 6)$

60) $(\log_a x - \log_a y) + 2\log_a z$

Solve the equation.

61) $2 + \log_3(2x + 5) - \log_3 x = 4$

62) $2^{(7 + 3x)} = \frac{1}{4}$

63) $\log_2 x = 3$

64) $3 \cdot 5^{2t - 1} = 75$

65) $\log_3 (x + 2) = 3 + \log_3 (x - 1)$

66) $\log_8 (3x + 3) = \log_8 (3x + 7)$

Solve the problem.

67) Sketch the graph of $y = 3^{-x} - 3$. State the domain and range.

Answer the following

68) Let $\log(a) = 3$ and $\log(b) = -12$. a) Find $\log (\frac{a}{b})$; b) $\log (a^2b^3)$.

Solve the problem.

69) $f(x) = 2^{x + 2}$ and $g(x) = 2^{-x + 4}$.
Find the point of intersection of the graphs of f and g by solving $f(x) = g(x)$.

70) The growth in population of a city can be seen using the formula $p(t) = 4055e^{0.002t}$, where t is the number of years since 1933. Use this formula to calculate in which year will the population reach 200,000?

7. OBJECTIVE: Trigonometry

Name the quadrant in which the angle θ lies.

71) $\tan \theta > 0$, $\sin \theta < 0$

Name the quadrant in which the angle θ lies.

72) $\sin \theta > 0$, $\cos \theta < 0$

73) $\csc \theta > 0$, $\sec \theta > 0$

In the problem, sin θ and cos θ are given. Find the exact value of the indicated trigonometric function.

74) $\sin \theta = \frac{1}{4}$, $\cos \theta = \frac{\sqrt{15}}{4}$ Find $\tan \theta$.

Find the exact value of the indicated trigonometric function of θ.

75) $\sec \theta = \frac{9}{8}$, θ in quadrant IV Find $\tan \theta$.

76) $\tan \theta = \frac{15}{8}$, $180° < \theta < 270°$ Find $\cos \theta$.

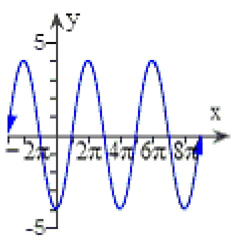
77) $\sin \theta = -\frac{2}{5}$, $\tan \theta > 0$ Find $\sec \theta$.

Answer the following

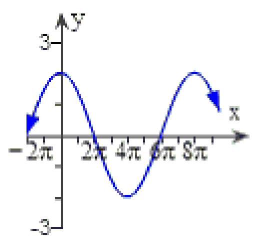
78) Let $f(x) = -4 \sin (\frac{1}{3} x)$. State the amplitude and period. Provide the graph of the f(x).

Choose the correct graph for:

79) $y = 2 \cos (\frac{1}{4} x)$



a



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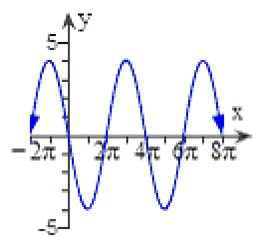
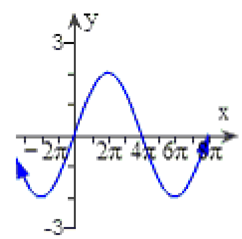


c



d

Establish the identity.

80) $\tan u(\csc u - \sin u) = \cos u$

81) $\cot \theta \cdot \sec \theta = \csc \theta$

82) $(\sec u - \tan u)(\sec u + \tan u) = 1$

83) $\csc^2 u - \cos u \sec u = \cot^2 u$

Find the exact value of the expression.

84) $\cos \frac{5\pi}{18} \cos \frac{2\pi}{9} - \sin \frac{5\pi}{18} \sin \frac{2\pi}{9}$

85) $\sin 10° \cos 110° + \cos 10° \sin 110°$

86) $\dfrac{\tan 160° - \tan 40°}{1 + \tan 160° \tan 40°}$

Find the exact value under the given conditions.

87) $\sin \alpha = \frac{15}{17}$, $0 < \alpha < \frac{\pi}{2}$; $\cos \beta = \frac{4}{5}$, $0 < \beta < \frac{\pi}{2}$ Find $\cos (\alpha + \beta)$.

88) $\sin \alpha = \frac{15}{17}, \frac{\pi}{2} < \alpha < \pi$; $\cos \beta = \frac{5}{13}, 0 < \beta < \frac{\pi}{2}$ Find $\sin(\alpha - \beta)$.

Use the information given about the angle θ, $0 \le \theta \le 2\pi$, to find the exact value of the indicated trigonometric function.

89) $\sin \theta = \frac{8}{17}, \quad 0 < \theta < \frac{\pi}{2}$ Find $\cos(2\theta)$.

Solve the problem.

90) A rain gutter is to be constructed of aluminum sheets 12 inches wide. After marking off a length of 4 inches from each edge, this length is bent up at an angle θ. See the figure below. The Area 'A' of the opening may be expressed as a function of θ as: $A(\theta) = 16 \sin \theta (\cos \theta + 1)$. Find the area for 30°.