Math120 - Precalculus. **Final Review**

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1 Algebra

1. Use the properties of exponents to simplify the following expression, writing your answer with only positive exponents.

   (a) $5^{-7}5^5$ (b) $((3z^{-3}y^2)^5)^{-1}$ (c) $(10x^3)^{-2}$ (d) $\left(\frac{5x^{-2}}{8y^{-2}}\right)^{-3}$

2. Factor the following polynomial.

   (a) $y^2 + 10y + 16$ (b) $x^4 + 2x^2 - 3$ (c) $t^3 - 8$ (d) $x^3 - 4x$

3. Solve the following equations. If needed, write your answer as a fraction reduced to lowest terms.

   (a) $-7(3w - 2) = 22(5 - w)$
   (b) $|5y + 7| - 8 = -5$
   (c) $y^2 + 10y + 24 = 0$
   (d) $x^2 - 4x = 45$
   (e) $7y^2 + 22y + 24 = 6y^2 + 36y - 21$
   (f) $\sqrt{9 - y} - y = 3$

4. Solve the following inequalities.Describe the solution set using interval notation first and then graph it.

   (a) $2 \leq \frac{y+1}{2} \leq 5$ (b) $4|t + 2| \leq 20$

5. Multiply or divide the following rational expressions, as indicated, and simplify your answer.

   (a) $\frac{x^2 - 12x + 27}{x + 3} \div \frac{x^2 + 3x - 18}{x + 3}$ (b) $\frac{4x - 4}{x} \div \frac{9x^2}{8x - 8}$

6. Find the restricted values of x for the rational expression (the domain) . If there are no restricted values of x, then state "No Restrictions".

   (a) $\frac{x^2 + x + 15}{x^3 - 4x}$ (b) $\frac{2x - 5}{x^2 - 81}$

7. Simplify the expression. Assume that all variables are positive when they appear.

   (a) $\sqrt{50} - \sqrt{18} - \sqrt{8}$
   (b) $\sqrt[3]{8a^6b^{10}c^{21}}$
   (c) $\sqrt[3]{-27x^{18}y^{36}}$
   (d) $(\sqrt{11} - \sqrt{8})(\sqrt{11} + \sqrt{8})$

8. Find the values of a, b, and c for which the quadratic equation $ax^2 + bx + c = 0$ has the given numbers as solutions. Then use those values to write a quadratic equation.

$$x_1 = \frac{1}{2}, x_2 = 2$$

2 Lines and Circles

9. Consider the following equation. $3y - 24 = 4x$

   (a) Determine the x- and y- intercepts of the given equation. If one of the intercepts does not exist, state "absent" for that intercept.

   (b) Graph the given equation by plotting the x and y intercepts. If an intercept does not exist, use another point to plot the graph.

10. Determine whether or not these points are the vertices of a right angled triangle:$(3, -5), (9, -5), (9, 0)$.

11. For the points $A = (-6, -1)$ and $B = (2, -10)$.

    (a) Find the distance between A and B.

    (b) Find the coordinates of the midpoint.

    (c) Determine the slope of the line that passes through A and B. Please enter your answer in simplest form. If the slope is undefined state "Undefined".

12. Find the slope of the line determined by the following equations. Please enter your answer in simplest form. If the slope is undefined state "Undefined".

    (a) $5y + 3x = 7$

    (b) $4y = 8$

    (c) $3x - 1 = 0$

    (d) $y = 4x - 1$

13. Consider the following equation. $x + 4y = 5$

    (a) Rewrite the equation in slope-intercept form.

    (b) Given $x = -7$, find the value for y and graph.

    (c) Given $x = -3$, find the value for y and use the points to complete the graph of the line.

14. Write the slope-intercept form of the equation for the line that passes through the points $(-6, 3)$ and $(1, 4)$.

15. Consider the following equations of two lines. Reduce all fractions to lowest terms. $6 - \frac{2y-5x}{2} = 5x + 4$ and $5x - 2y = 10$

    (a) Rewrite the first equation in slope-intercept form.

    (b) Rewrite the second equation in slope-intercept form.

    (c) Determine if these two lines are perpendicular.

16. Consider the following equation of a line. Reduce all fractions to lowest terms. $8x + 4y = 15$

    (a) Rewrite this equation in slope-intercept form.

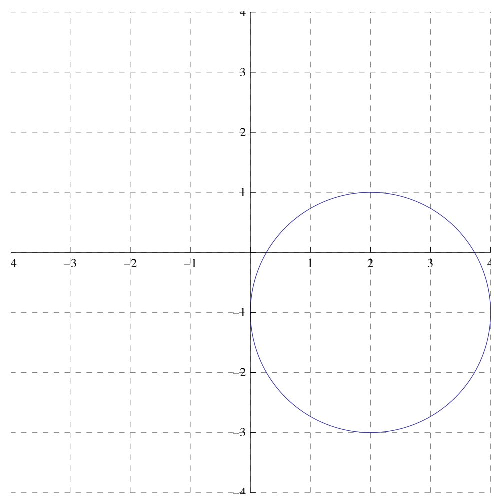
    (b) Find the equation, in slope-intercept form, for the line which is parallel to this line and passes through the point $(-7, 8)$.

17. Complete the sentences below:
    The line $y = 5x + 2$ and $y = ax - 1$ are **perpendicular** if $a =$ ______________________.
    The line $y = 3x - 1$ and $y = ax$ are **parallel** if $a =$ ______________________.

18. The slope of a **vertical** line is ________________ ; the slope of a **horizontal** line is ________________.

19. Find the standard form of the equation for the circle with radius 3 and center $(3, 1)$

20. Consider the equation $x^2+y^2-8y+7=0$, Find the center (h,k) ,and radius, r of this circle and graph the circle.
21. Consider the equation $x^2+y^2-14x+10y+38=0$, Find the center (h,k) ,and radius, r of this circle and graph the circle.
22. Consider the circle pictured below.
    (a) Find the center (h,k) ,and radius, r of this circle.
    (b) Write the equation of the circle in standard form.
    (c) Find the intercepts.



3 Quadratic Functions

23. Answer each question about the function $f(x)=x^2-2x-1$
    (a) Is the point $(2,-9)$ on the graph of f?
    (b) If $f(x)=-1$, what is x?
    (c) List the vertex and the axis of symmetry of f.
    (d) List the x-intercepts if any of the graph of f.
    (e) List the y-intercepts if any of the graph of f.
    (f) Graph $f(x)$.
24. Consider the following quadratic function. $k(x)=(x-2)^2-9$
    (a) Determine the x-intercept(s), if any, and the y-intercepts of this function as ordered pair(s).
    (b) Determine the vertex and the axis of symmetry.
    (c) Graph this quadratic function by identifying two other points on the parabola.
25. Determine the equation of the quadratic function whose vertex is $(-1,4)$ and the y-intercept is -3.
26. If $(b,15)$ is a point on the graph of the function $y=x^2+5x+1$, what is b?

4 Functions and Graphs

27. Choose the domain of the function $f(x)=\dfrac{x}{\sqrt{1-x^2}}$.

(a) $(-\infty, \infty)$
(b) $(-1, 1)$
(c) $(-1, 1]$
(d) $(-\infty, -1) \cup (1, \infty)$
(e) $[-1, 1]$
(f) $(-\infty, -1] \cup [1, \infty)$

28. Given the following function: $h(x) = \frac{5-3x}{x-2}$, determine the domain of $h(x)$. Express your answer in interval notation.

29. Determine the domain and range of the function defined as $g(x) = \sqrt{x+7}$. Express your answer in interval notation.

30. For $f(x) = x^2 + 3$ evaluate and simplify:

    (a) $f(x+1) =$
    (b) $f(x+h) =$
    (c) $f(x+h) - f(x) =$
    (d) $\dfrac{f(x+h) - f(x)}{h} =$

31. Determine if the following function is even, odd, or neither.

    (a) $f(x) = (x+3)^2 + 5$
    (b) $h(x) = -\dfrac{x^3}{4}$
    (c) $g(x) = \dfrac{x^2}{\cos x + 1}$
    (d) $i(x) = \dfrac{x}{x^2 + 2}$

32. For $f(x) = x^3 + x$ and $g(x) = x^2 + 3$ determine

    (a) $(f \circ g)(1)$.
    (b) $(g \circ f)(2)$.

33. For $f(x) = x + 5$ and $g(x) = x^2 - 1$ find the formula for

    (a) $(f + g)(x)$.
    (b) $(\frac{f}{g})(x)$.
    (c) $(fg)(x)$.
    (d) $(f - g)(x)$.
    (e) $(f \circ g)(x)$.
    (f) $(g \circ f)(x)$.

34. Find functions f and g such that $(f \circ g)(x) = |5x + 1|$

35. Evaluate the following for $f(x)$ if $f(x) = \begin{cases} |x|, & x \leq -2 \\ x+2, & -2 < x < 4 \\ x^3, & x \geq 4 \end{cases}$,

    (a) $f(5)$
    (b) $f(-2)$
    (c) $f(0)$
    (d) $f(-3)$

36. Given the following relation: $y = 2x + 5$

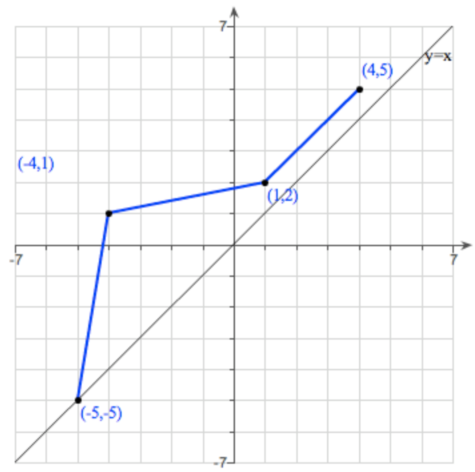
    (a) Enter four points for the inverse of the above relation.
    (b) Find the inverse.
    (c) Enter the domain and range of the inverse.

37. Find a formula for the inverse of the given function.

    (a) $g(x) = x^{\frac{1}{3}} + 1$
    (b) $p(x) = \dfrac{-x-2}{x-5}$

38. Use the given graph of $y = f(x)$ to evaluate the following and graph the inverse of $f(x)$.

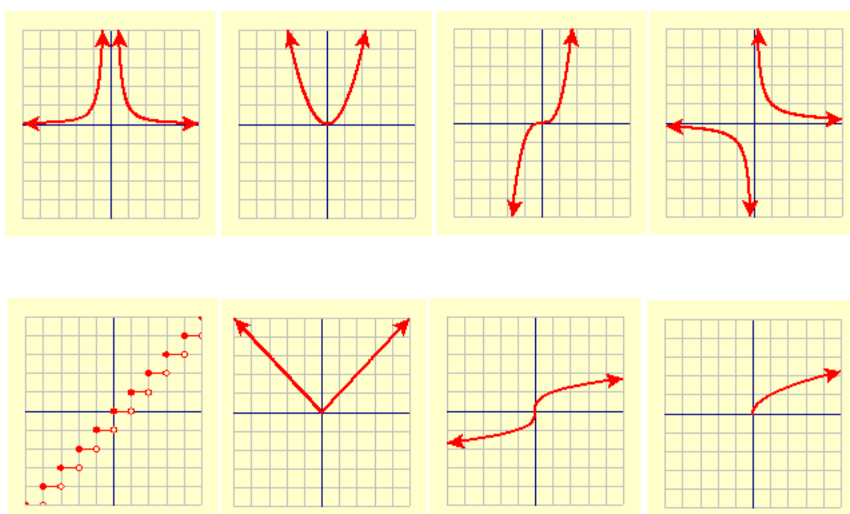
(a) $f(-5)$ (b) $f(3)$ (c) $f^{-1}(1)$ (d) $f^{-1}(-5)$



39. Consider the following function. $h(x) = \sqrt{3-x} - 1$

(a) Identify the more basic function that has been shifted, reflected, stretched, or compressed.

(b) Indicate the shape of the function that was found in step 1.



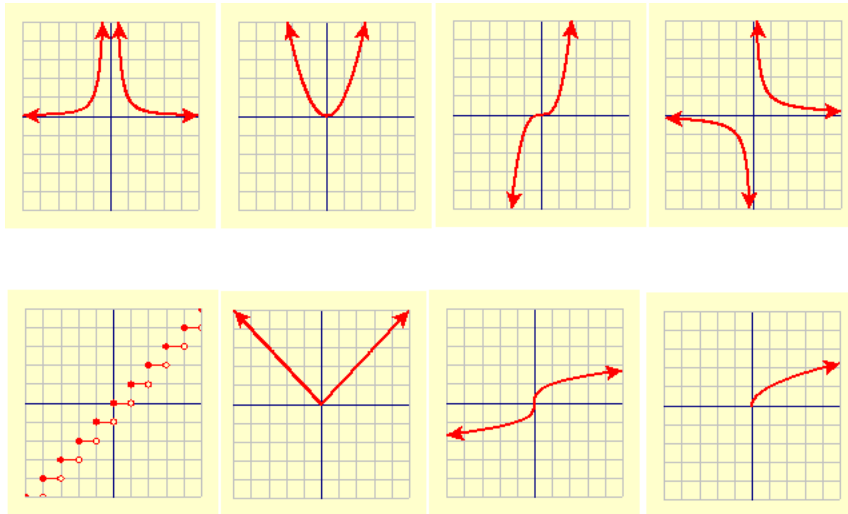
(c) Graph this function by indicating how the basic function found in step 1 has been shifted, reflected, stretched, or compressed.

(d) Determine the domain and range of this function. Write your answer in interval notation.

40. Consider the following function. $f(x) = \dfrac{1}{x-4} - 5$

(a) Identify the more basic function that has been shifted, reflected, stretched, or compressed.

(b) Indicate the shape of the function that was found in step 1.

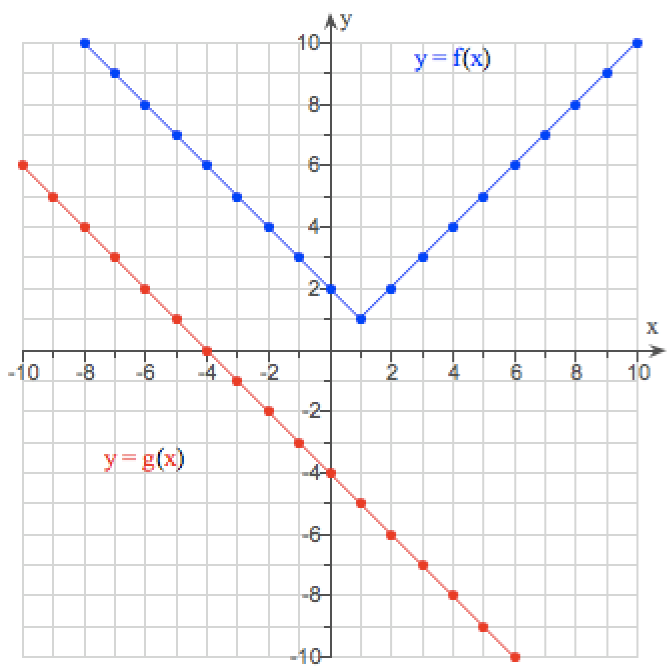


(c) Graph this function by indicating how the basic function found in step 1 has been shifted, reflected, stretched, or compressed.

(d) Determine the domain and range of this function. Write your answer in interval notation or symbol notation.

(e) Identify the horizontal, vertical and the oblique asymptotes if any of this function.

41. For the graph shown below determine:



(a) $f(-3) =$

(b) $g(2) =$

(c) $(f+g)(4) =$

(d) $(fg)(-6) =$

(e) $(g \circ f)(2) =$

(f) $(f \circ g)(1) =$

42. Evaluate each expression using the values in the given table:

x	−3	−2	−1	0	1	2	3
f(x)	−9	−7	−5	−3	−1	1	3
g(x)	3	2	1	0	−1	−2	−3

(a) $(f \circ g)(3) =$

(b) $(g \circ f)(2) =$

(c) $(f \circ f)(0) =$

(d) $(g \circ g)(-1) =$

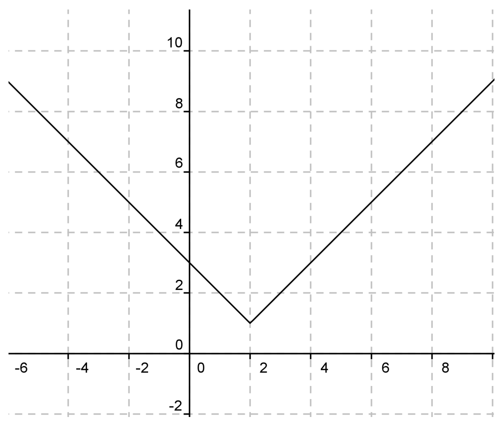
43. Choose the function that matches the given graph.

(a) $y = -2x^2$

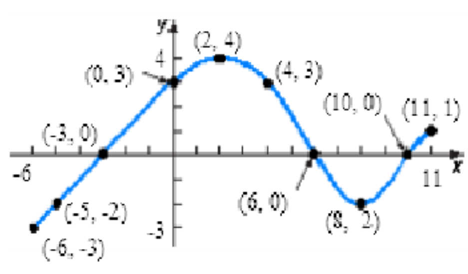
(b) $y = -|x+1|$

(c) $y = |x+1| + 2$

(d) $y = |x-2| + 1$



44. For the fuction shown below



(a) What is $f(2)$?

(b) Is $f(-2)$ positive or negative ?

(c) For what values x is $f(x) < 0$?

(d) Domain?

(e) Range?

(f) Intercepts?

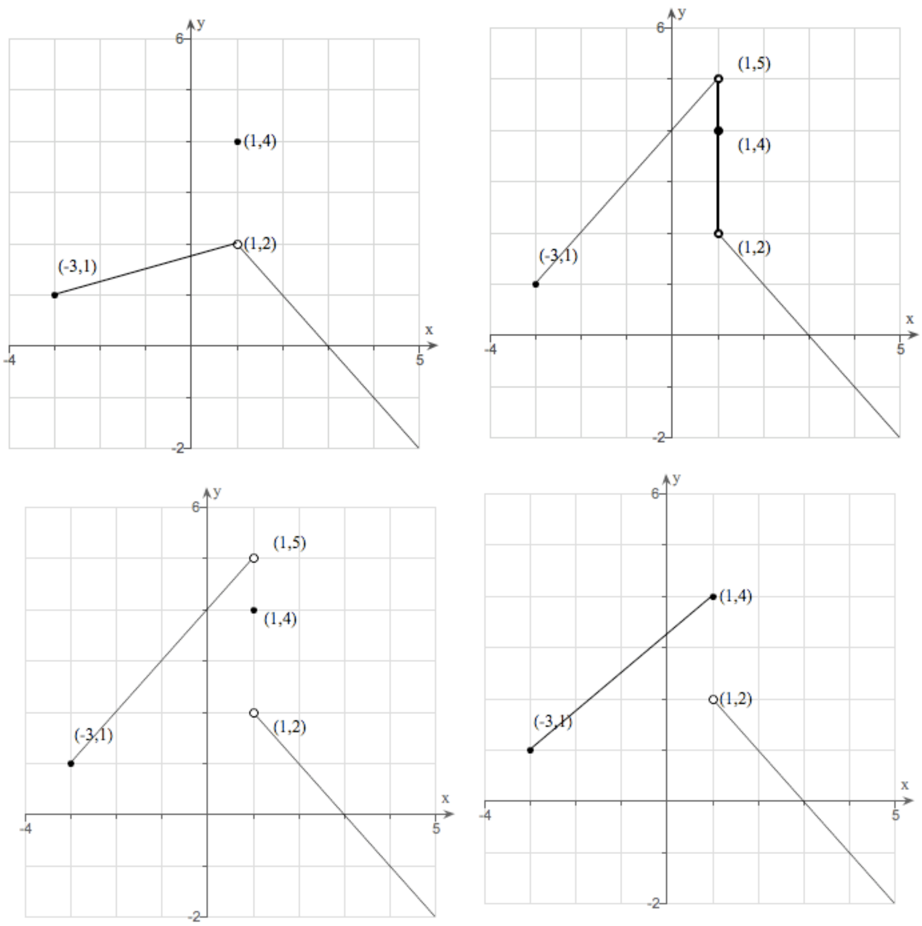
45. A function$f$ has an inverse function. If the graph of f^{-1} lies in quadrant III, in which quadrant does the graph of $f(x)$ lies?

46. [8 Points] If $f(x) = \begin{cases} x+4, & -3 \le x < 1 \\ 4, & x = 1 \\ -x+3, & x > 1 \end{cases}$,

(a) Evaluate the following

i. $f(-2) =$ ii. $f(0) =$ iii. $f(1) =$ iv. $f(4) =$

(b) Choose the correct graph of this function below.



5 Polynomial and Rational Functions

47. Use polynomial long division to rewrite the following fraction in the form $q(x) + \frac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

$$\frac{x^3 + 4x^2 - 21x - 13}{x - 4}$$

48. Use synthetic division or long division to determine if $k = 5$ is a zero of this polynomial. If not, determine $p(k)$.

$$p(x) = 3x^4 - 19x^3 - 6x^2 + 142x - 60$$

49. Given the following polynomial: $q(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$

   (a) Identify the potential rational zeros.

   (b) Use polynomial division or synthetic division and the quadratic formula, if necessary, to identify the actual zeros.

50. Given the following rational function: $f(x) = \dfrac{-14x^2 + 27x - 9}{7x - 3}$

   (a) Find equations for the vertical asymptotes, if any, for the rational function.

   (b) Find equations for the horizontal or oblique asymptotes, if any, for the rational function.

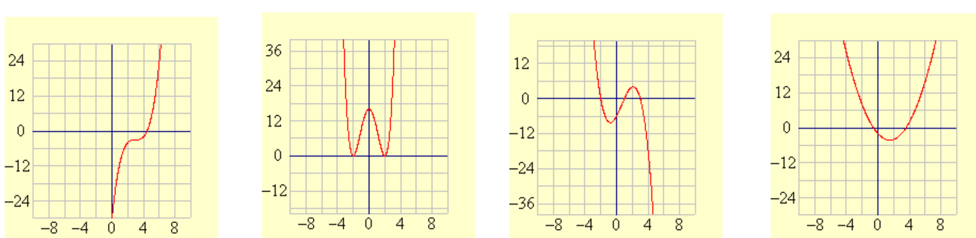
   (c) Find the domain of the rational function.

51. Construct a polynomial function that has Second-degree, with zeros of -4 and 3, and goes to $-\infty$ as $x \to -\infty$.

52. Construct a polynomial function that has degree three, with zeros of -4 with multiplicity 2, and 3 with multiplicity 1, and goes to $-\infty$ as $x \to \infty$.

53. Solve the polynomial equation $x^4 - 6x^2 + 8 = 0$, by factoring or using the quadratic formula, making sure to identify all the solutions.

54. Match the polynomial function $z(x) = (x - 1)(x + 2)(3 - x)$, by determining the x-intercepts, the y-intercept, and the behavior as $x \to \infty$ from one of the graphs labeled below.



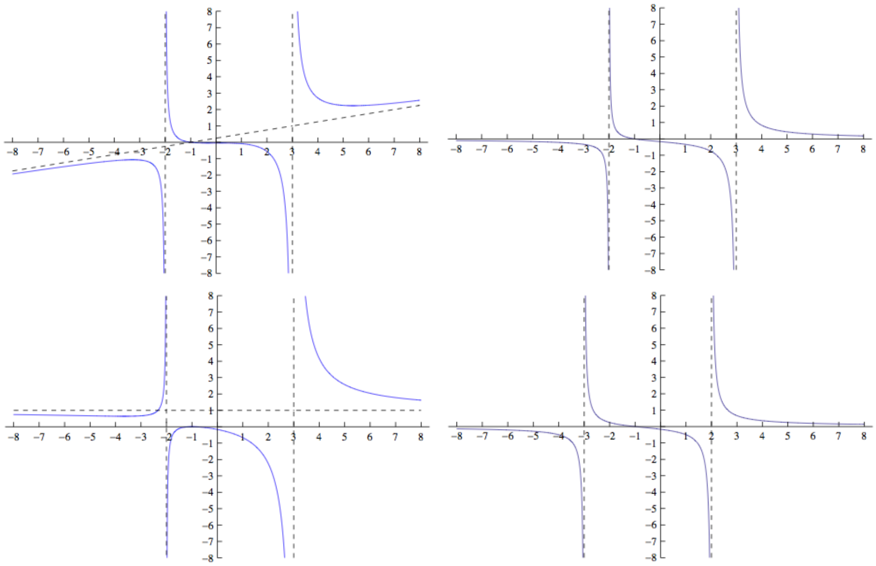
55. Sketch a graph of the function $Q(x) = (x - 2)^2(x + 1)(x - 5)$.

56. Given the rational function: $f(x) = \dfrac{x + 1}{x^2 - x + 6}$

   (a) Find equations for the vertical asymptotes, if any, for the rational function.

   (b) Find equations for the horizontal or oblique asymptotes, if any, for the rational function.

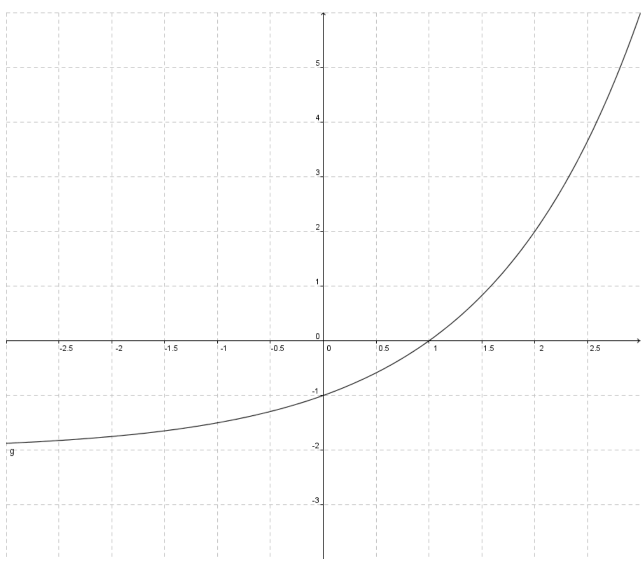
   (c) Which of the following graphs is the graph of $f(x)$?



6 Exponential Functions

57. Which function matches the graph shown in the following graph ?

(a) $y = 2^{x+2}$ (b) $y = 2^{x+1} + 2$ (c) $y = 2^{x-2}$ (d) $y = 2^{x} - 2$



58. Use the properties of logarithms to expand or simplify the following expression as much as possible. Simplify any numerical expressions that can be evaluated without a calculator.

(a) $\log_9(81x^3)$ (c) $\log_3(\frac{x-4}{x^7})$ (e) $\log_{10} 5 + \log_{10} 2$

(b) $\ln \sqrt[5]{ey}$ (d) $\ln e^{42}$ (f) $e^{\ln 25}$

59. Let $\log(a) = 3$ and $\log(b) = -12$. Find $\log(\frac{a}{b})$ and $\log(a^2b^3)$.

60. Solve the following equations. If there is no solution, state "No Solution".

(a) $(\frac{1}{2})^{5x+5} = (\frac{1}{4})^4$

(b) $3e^{4x} = 90$

(c) $\log_9(x^2 + 12x + 32) - \log_9(x + 8) = 0$

(d) $e^{2x+5} = 12^{\frac{2x}{7}}$

(e) $\log_5(x - 1) + \log_5(x - 3) = 1$

(f) $5^{-x-9} = 625$

(g) $2^{x^2+5x} = 4^{-3}$

(h) $(\frac{1}{3})^{3x+5} = 9^x$

61. Find $f \circ g(x)$ and $g \circ f(x)$ when $f(x) = \ln(x)$ and $g(x) = e^{4x}$.

62. Find the domain of the function $f(x) = \ln(x - 3)$. Determine the range and any asymptotes of $f(x)$.

63. For $f(x) = 2 + \log(x - 5)$.

(a) Identify and graph the more basic function that has been shifted, reflected, stretched, or compressed to obtain $f(x)$.

(b) Graph $f(x)$.

7 Trigonometric Functions

64. Convert the radian measure to degrees, or the degree measure to radians.

(a) $\frac{3\pi}{2}$

(b) $630°$

(c) $\frac{11\pi}{6}$

(d) $270°$

65. Name the quadrant in which the angle θ lies when $\cos\theta < 0$ and $\tan\theta < 0$.

66. Use trigonometric identities to simplify the expression.

(a) $\sec x \cos x$

(b) $\dfrac{1}{\sec^2\theta - 1}$

(c) $\dfrac{\sec\theta}{\csc\theta}$

(d) $\csc(x + 2\pi)\sin x$

(e) $\dfrac{\sin(\beta)\tan(\frac{\pi}{2} - \beta)}{\cos(\beta)}$

(f) $\cos(\dfrac{5\pi}{6} - \dfrac{7\pi}{6})$

67. If $\sin\theta = \frac{1}{3}$ and θ is in quadrant II, find all other trigonometric functions of θ.

68. Find the exact values of each of the remaining trigonometric functions of θ when $\tan\theta = -\frac{1}{8}$ and $\sec\theta < 0$.

69. Find the exact values of the given expression using the table of basic trigonometric values.

(a) $\tan(11\pi)$

(b) $\sec(20\pi)$

(c) $\sin(\frac{5\pi}{4})$

(d) $\cot(\frac{9\pi}{4})$

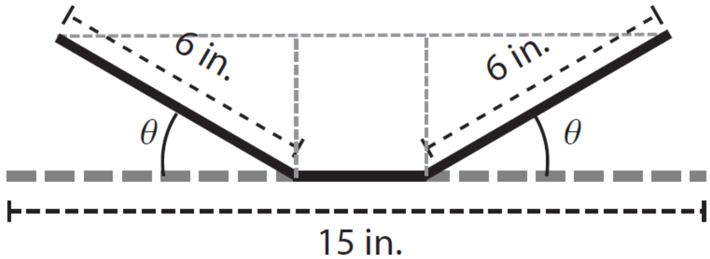
(e) $\cot(420°)$

(f) $\tan(\frac{13\pi}{4})$

(g) $\cos(\frac{13\pi}{6})$

(h) $\csc(\frac{9\pi}{6})$

70. A rain gutter is to be constructed of aluminum sheets that are 15 inches wide. After marking off length of 6 inches from each edge, the sides are each bent up at an angle of θ.



(a) Express the area of the opening as a function of θ.

(b) Find the area for $\theta = \frac{\pi}{6}$, and $\theta = \frac{\pi}{4}$.

71. Use the sum and difference identities to rewrite the following expression as a trigonometric function of a single number.

(a) $\dfrac{\tan 70 + \tan 45}{1 - \tan 70 \tan 45}$

(b) $\cos(\frac{\pi}{6})\cos(\frac{3\pi}{5}) + \sin(\frac{\pi}{6})\sin(\frac{3\pi}{5})$

(c) $\dfrac{\tan\frac{5\pi}{14} + \tan\frac{2\pi}{14}}{1 - \tan\frac{5\pi}{14}\tan\frac{2\pi}{14}}$

(d) $\sin(120)\cos(30) + \cos(120)\sin(30)$

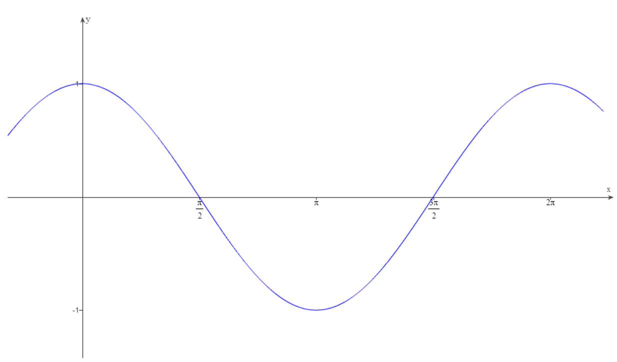
(e) $\cos(\frac{\pi}{4})\sin(\frac{\pi}{3}) + \sin(\frac{\pi}{4})\cos(\frac{\pi}{3})$

72. Determine the amplitude and the period of the function without graphing.

(a) $y = -5\cos(6x)$ (b) $y = 3\sin(\pi x)$ (c) $y = \frac{1}{3}\sin(2x)$ (d) $y = \cos(\frac{x}{\pi})$

73. Which function matches the graph shown in the following graph ?

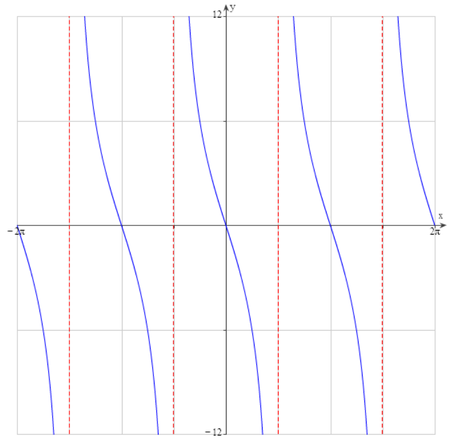
(a) $y = \cos x$ (b) $y = \cos 2x$ (c) $y = \sin 2x$ (d) $y = \sin x$



74. Find $f \circ g(x)$ and $g \circ f(x)$ when $f(x) = \cos(x)$ and $g(x) = -6x$.

75. Which function matches the graph shown in the following graph ?

(a) $y = -\tan x$ (b) $y = \tan 6x$ (c) $y = -\cot x$ (d) $y = \cot 6x$



76. Use the sum and difference identities to determine the exact value of the expression $\sin(-\frac{11\pi}{6})$

77. If $\sin\alpha = -\frac{12}{13}$ and α is in quadrant III and $\sin\beta = \frac{24}{25}$ and β is in quadrant II. Find $\cos(\alpha - \beta)$.

78. If $\cos\alpha = \frac{8}{17}$ and α is in quadrant IV and $\cos\beta = -\frac{8}{17}$ and β is in quadrant II. Find $\sin(\alpha + \beta)$.

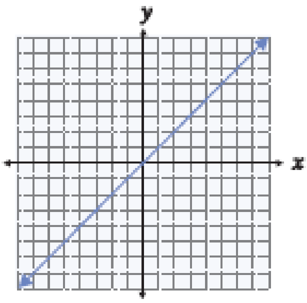
79. Determine $\cos 2x$ if $\sin x = \frac{3}{5}$ and $\cos x$ is positive.

80. Use trigonometric identities, to solve the following trigonometric equation on the interval $[0, 2\pi]$.

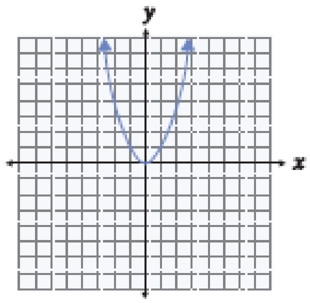
    (a) $5\cos(-x) = 3\cos(x) + 1$

    (b) $2\sin^2 x - 1 = 0$

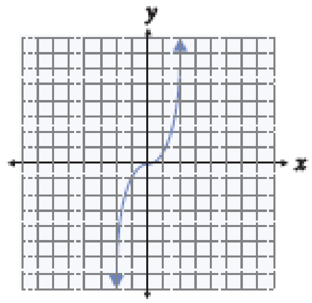
common graphs



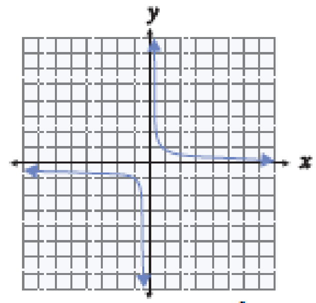
The function $f(x) = x$



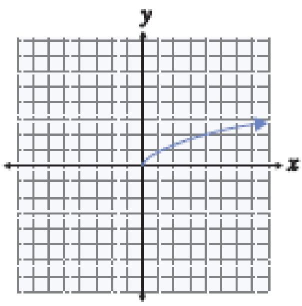
The function $f(x) = x^2$



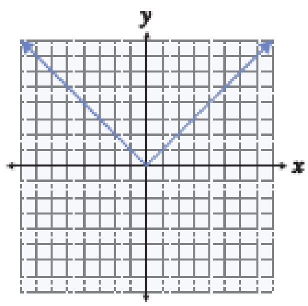
The function $f(x) = x^3$



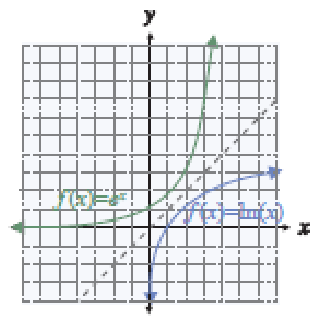
The function $f(x) = \frac{1}{x}$



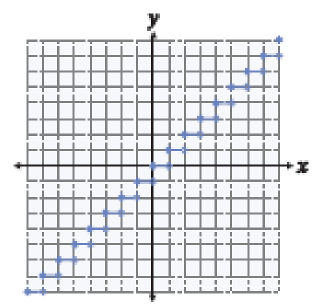
The function $f(x) = \sqrt{x}$ or $x^{1/2}$



The function $f(x) = |x|$



The functions e^x and $\ln(x)$



The function $f(x) = [\![x]\!]$

properties of exponents and radicals

$a^m \cdot a^n = a^{m+n}$ $\quad (a^n)^m = a^{nm}$

$\frac{a^n}{a^m} = a^{n-m}$ $\quad (ab)^n = a^n b^n$

$a^{-n} = \frac{1}{a^n}$ $\quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

$(a)^{1/n} = \sqrt[n]{a}$ $\quad (a)^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$

special product formulas

$A^2 - B^2 = (A - B)(A + B)$

$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

the quadratic formula

The solutions of the equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

properties of logarithms

For $a > 0$, a is not equal to 1, $x, y > 0$ and r is a real number:

$\log_a(x) = y$ and $x = a^y$ are equivalent

$\log_a(1) = 0$

$\log_a(a) = 1$

$\log_a(a^x) = x$

$a^{\log_a(x)} = x$

$\log_a(xy) = \log_a(x) + \log_a(y)$

$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

$\log_a(x^r) = r \log_a x$

change of base formula

$a, b, x > 0;\ a, b \neq 1;$

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

midpoint formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

radian and degree measure

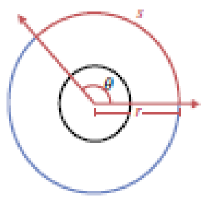
$180° = \pi$

$1° = \frac{\pi}{180}$ $\qquad$ $\left(\frac{180}{\pi}\right)^{\circ} = 1$

$x° = (x)\left(\frac{\pi}{180}\right)$ rad $\qquad$ $x \text{ rad} = x\left(\frac{180}{\pi}\right)^{\circ}$

$s = \left(\frac{\theta}{2\pi}\right)(2\pi r)$

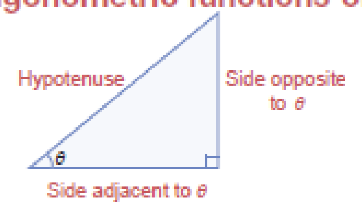
$= r\theta$



$A = \left(\frac{\theta}{2\pi}\right)(\pi r^2) = \frac{r^2\theta}{2}$

$\omega = \frac{\theta}{t}$ $\qquad$ $v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$

trigonometric functions of acute angles

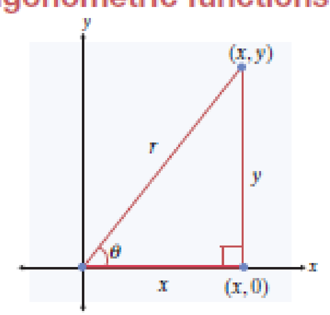


$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\qquad$ $\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hyp}}{\text{opp}}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\qquad$ $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{hyp}}{\text{adj}}$

$\tan\theta = \frac{\text{opp}}{\text{adj}}$ $\qquad$ $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{adj}}{\text{opp}}$

trigonometric functions of any angle



$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y}$ (for $y \neq 0$)

$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x}$ (for $x \neq 0$)

$\tan\theta = \frac{y}{x}$ (for $x \neq 0$) $\qquad$ $\cot\theta = \frac{x}{y}$ (for $y \neq 0$)

cofunction identities

$\sin x = \cos\left(\frac{\pi}{2} - x\right)$ $\qquad$ $\csc x = \sec\left(\frac{\pi}{2} - x\right)$

$\cos x = \sin\left(\frac{\pi}{2} - x\right)$ $\qquad$ $\sec x = \csc\left(\frac{\pi}{2} - x\right)$

$\cot x = \tan\left(\frac{\pi}{2} - x\right)$ $\qquad$ $\tan x = \cot\left(\frac{\pi}{2} - x\right)$

reciprocal identities

$\csc x = \frac{1}{\sin x}$ $\qquad$ $\sec x = \frac{1}{\cos x}$ $\qquad$ $\cot x = \frac{1}{\tan x}$

quotient identities

$\tan x = \frac{\sin x}{\cos x}$ $\qquad$ $\cot x = \frac{\cos x}{\sin x}$

period identities

$\sin(x + 2\pi) = \sin(x)$ $\qquad$ $\csc(x + 2\pi) = \csc(x)$

$\cos(x + 2\pi) = \cos(x)$ $\qquad$ $\sec(x + 2\pi) = \sec(x)$

$\tan(x + \pi) = \tan(x)$ $\qquad$ $\cot(x + \pi) = \cot(x)$

even/odd identities

$\sin(-x) = -\sin x$ $\qquad$ $\csc(-x) = -\csc x$

$\cos(-x) = \cos x$ $\qquad$ $\sec(-x) = \sec x$

$\tan(-x) = -\tan x$ $\qquad$ $\cot(-x) = -\cot x$

pythagorean identities

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\tan^2 x + 1 = \sec^2 x$ $\qquad$ $1 + \cot^2 x = \csc^2 x$

commonly encountered angles

θ	Radians	Sin θ	Cos θ	Tan θ
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	—
180°	π	0	–1	0
270°	$\frac{3\pi}{2}$	–1	0	—

sum and difference identities

$$\sin(u+v)=\sin u\cos v+\cos u\sin v$$

$$\sin(u-v)=\sin u\cos v-\cos u\sin v$$

$$\cos(u+v)=\cos u\cos v-\sin u\sin v$$

$$\cos(u-v)=\cos u\cos v+\sin u\sin v$$

$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v}$$

$$\tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$$

double-angle identities

$$\sin 2u=2\sin u\cos u$$

$$\tan 2u=\frac{2\tan u}{1-\tan^2 u}$$

$$\cos 2u=\cos^2 u-\sin^2 u=2\cos^2 u-1=1-2\sin^2 u$$

power-reducing identities

$$\sin^2 x=\frac{1-\cos 2x}{2} \qquad \cos^2 x=\frac{1+\cos 2x}{2}$$

$$\tan^2 x=\frac{1-\cos 2x}{1+\cos 2x}$$

half-angle identities

$$\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}} \qquad \cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2}=\frac{1-\cos x}{\sin x}=\frac{\sin x}{1+\cos x}$$

product-to-sum identities

$$\sin x\cos y=\frac{1}{2}\left[\sin(x+y)+\sin(x-y)\right]$$

$$\cos x\sin y=\frac{1}{2}\left[\sin(x+y)-\sin(x-y)\right]$$

$$\sin x\sin y=\frac{1}{2}\left[\cos(x-y)-\cos(x+y)\right]$$

$$\cos x\cos y=\frac{1}{2}\left[\cos(x+y)+\cos(x-y)\right]$$

sum-to-product identities

$$\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x-\sin y=2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

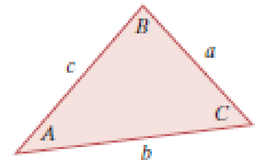
$$\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x-\cos y=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

the laws of sines and cosines

the law of sines

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$



the law of cosines

$$a^2=b^2+c^2-2bc\cos A$$

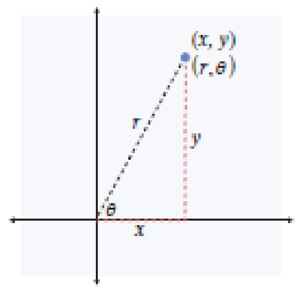
$$b^2=a^2+c^2-2ac\cos B$$

$$c^2=a^2+b^2-2ab\cos C$$

area of a triangle (sine formula)

$$\text{Area} = \frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$$

polar coordinates



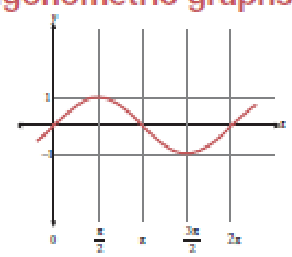
$$x=r\cos\theta$$

$$y=r\sin\theta$$

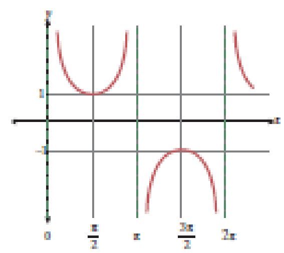
$$r^2=x^2+y^2$$

$$\tan\theta=\frac{y}{x} \quad (x\neq 0)$$

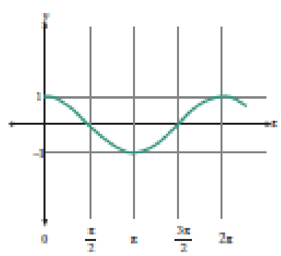
trigonometric graphs



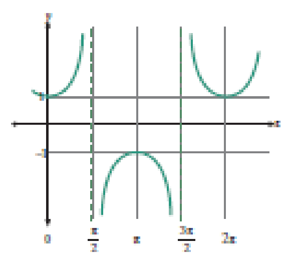
$y=\sin x$



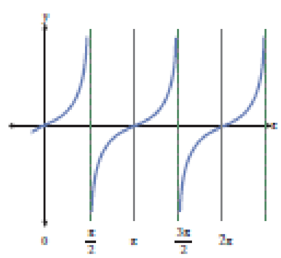
$y=\csc x$



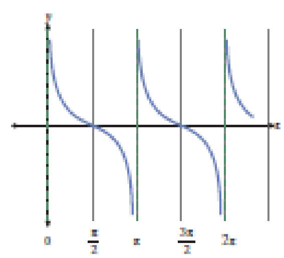
$y=\cos x$



$y=\sec x$



$y=\tan x$



$y=\cot x$