Circles

Standard Form of a Circle

The **standard form** of the equation for a circle of radius r with center (h, k) is

$$(x-h)^2+(y-k)^2=r^2.$$

THEOREM

The standard form of an equation of a circle of radius r with center at the origin $(0, 0)$ is

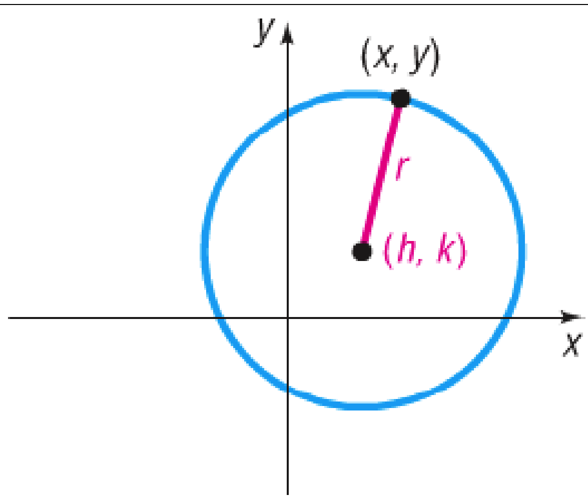
$$x^2 + y^2 = r^2$$

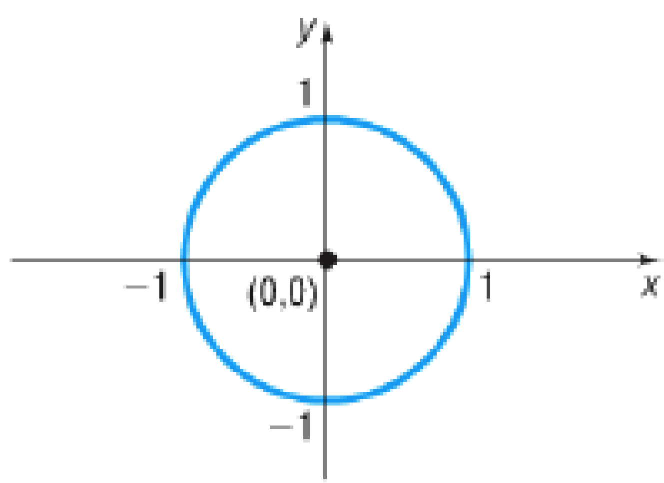
DEFINITION

If the radius $r = 1$, the circle whose center is at the origin is called the **unit circle** and has the equation

$$x^2 + y^2 = 1$$

See Figure 50. Notice that the graph of the unit circle is symmetric with respect to the x-axis, the y-axis, and the origin.





Writing the Standard Form of the Equation of a Circle

Write the standard form of the equation of the circle with radius 5 and center $(-3, 6)$.

Using equation (1) and substituting the values $r = 5$, $h = -3$, and $k = 6$, we have

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x + 3)^2 + (y - 6)^2 = 25$$

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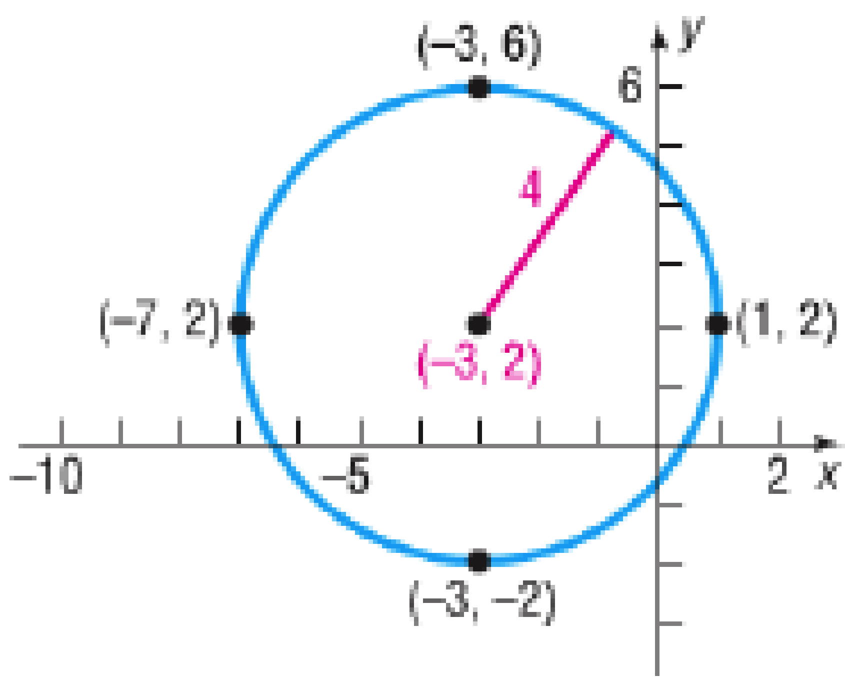
Graph the equation: $(x + 3)^2 + (y - 2)^2 = 16$

Since the equation is in the form of equation (1), its graph is a circle. To graph the equation, compare the given equation to the standard form of the equation of a circle. The comparison yields information about the circle.

$$(x + 3)^2 + (y - 2)^2 = 16$$
$$(x - (-3))^2 + (y - 2)^2 = 4^2$$
$$\uparrow \qquad \uparrow \qquad \uparrow$$
$$(x - h)^2 + (y - k)^2 = r^2$$

We see that $h = -3$, $k = 2$, and $r = 4$. The circle has center $(-3, 2)$ and a radius of 4 units. To graph this circle, first plot the center $(-3, 2)$. Since the radius is 4, we can locate four points on the circle by plotting points 4 units to the left, to the right, up, and down from the center. These four points can then be used as guides to obtain the graph. See Figure 51.

Figure 51



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Work with the General Form of the Equation of a Circle

If we eliminate the parentheses from the standard form of the equation of the circle given in Example 2, we get

$$(x + 3)^2 + (y - 2)^2 = 16$$
$$x^2 + 6x + 9 + y^2 - 4y + 4 = 16$$

which, upon simplifying, is equivalent to

$$x^2 + y^2 + 6x - 4y - 3 = 0 \qquad \textbf{(2)}$$

It can be shown that any equation of the form

$$x^2 + y^2 + ax + by + c = 0$$

has a graph that is a circle, or a point, or has no graph at all. For example, the graph of the equation $x^2 + y^2 = 0$ is the single point $(0, 0)$. The equation $x^2 + y^2 + 5 = 0$, or $x^2 + y^2 = -5$, has no graph, because sums of squares of real numbers are never negative.

When its graph is a circle, the equation

$$x^2 + y^2 + ax + by + c = 0$$

is referred to as the **general form of the equation of a circle.**

Graphing a Circle Whose Equation Is in General Form

Graph the equation $x^2 + y^2 + 4x - 6y + 12 = 0$

Group the terms involving x, group the terms involving y, and put the constant on the right side of the equation. The result is

$$(x^2 + 4x) + (y^2 - 6y) = -12$$

Next, complete the square of each expression in parentheses. Remember that any number added on the left side of the equation must also be added on the right.

$$(x^2 + 4x + 4) + (y^2 - 6y + 9) = -12 + 4 + 9$$

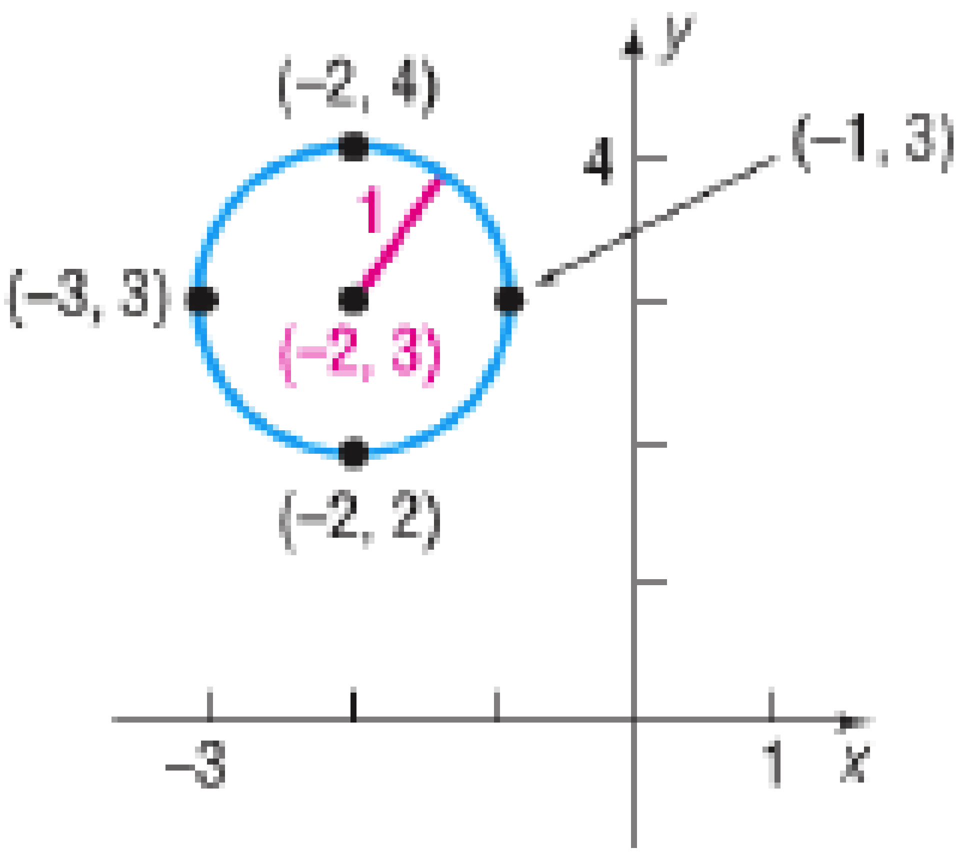
$$\left(\frac{4}{2}\right)^2 = 4 \qquad \left(\frac{-6}{2}\right)^2 = 9$$

$$(x + 2)^2 + (y - 3)^2 = 1 \quad \text{Factor.}$$

This equation is the standard form of the equation of a circle with radius 1 and center $(-2, 3)$.

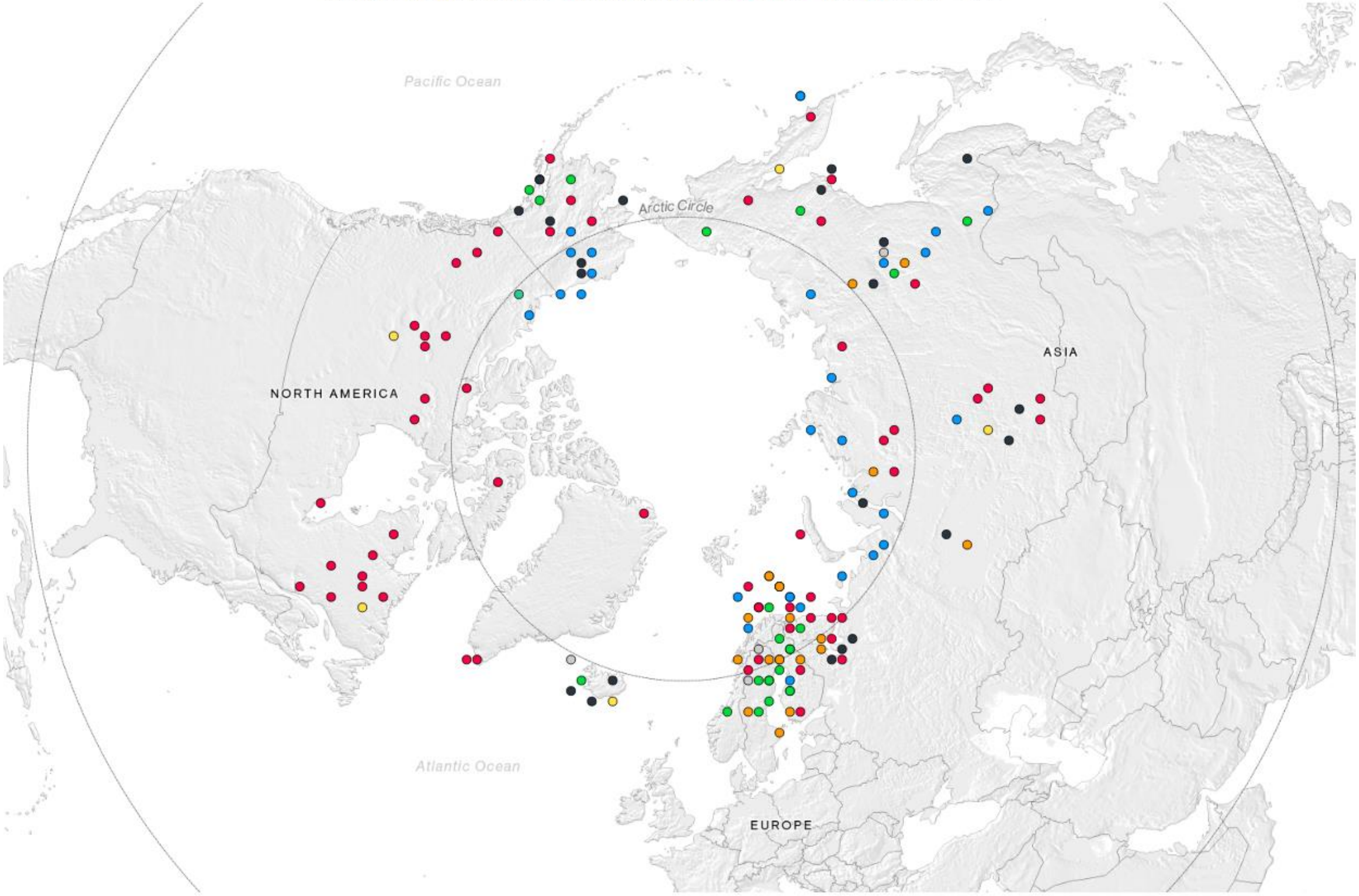
To graph the equation use the center $(-2, 3)$ and the radius 1. See Figure 52.

Figure 52



REAL LIFE EXAMPLE OF CIRCLES

Development in the Arctic
From roads to new mines, these are some of the largest projects on each country's wish list, according to Guggenheim Partners:
■ Transportation ■ Fossil Fuel Energy ■ Renewable/Nuclear Energy ■ Mining ■ Power ■ Economic ■ Civic



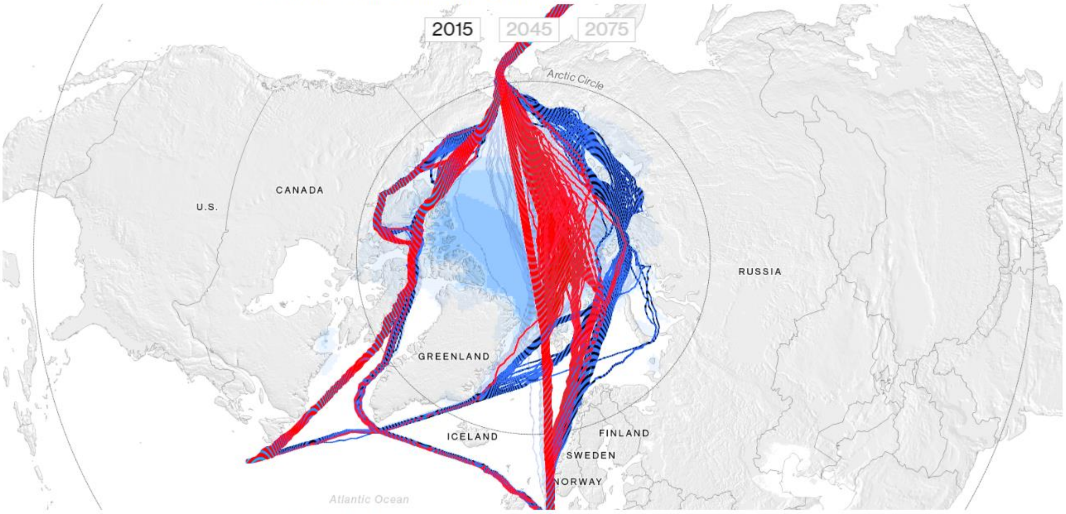
Potential Projects, and Their Estimated Cost, by Country

Source: Guggenheim Partners

As the Ice Melts, Arctic Shipping Routes Expand

■ Overwater shipping routes ■ Icebreaker shipping routes

Sea ice thickness: 0–0.5 0.5–1.0 1.0–1.5 1.5+ meters



Source: Melia, N., K. Haines, and E. Hawkins, *Sea Ice Decline and 21st Century Trans-Arctic Shipping Routes*

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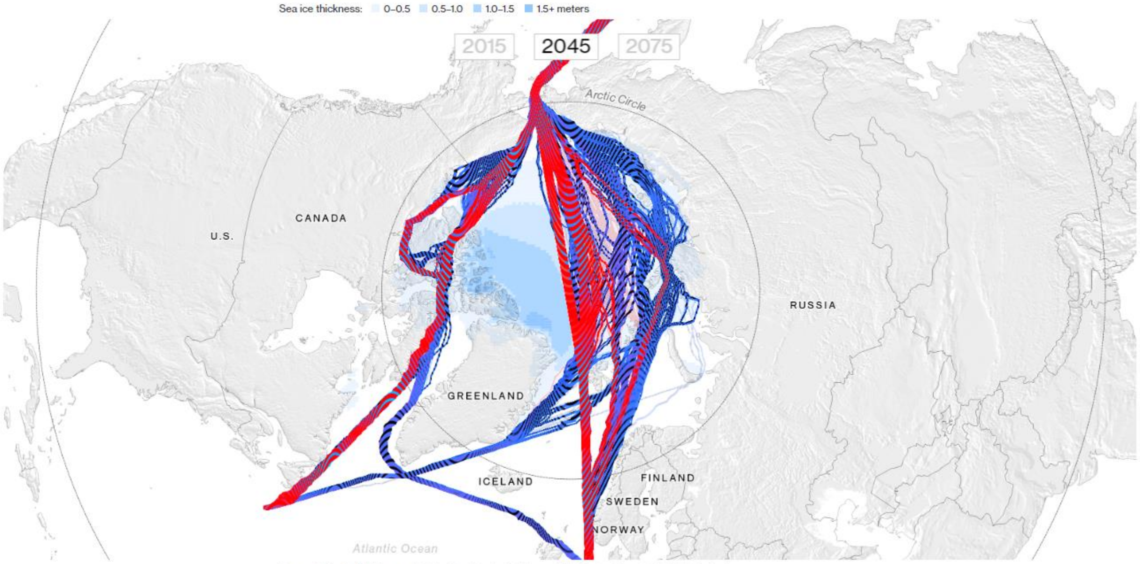

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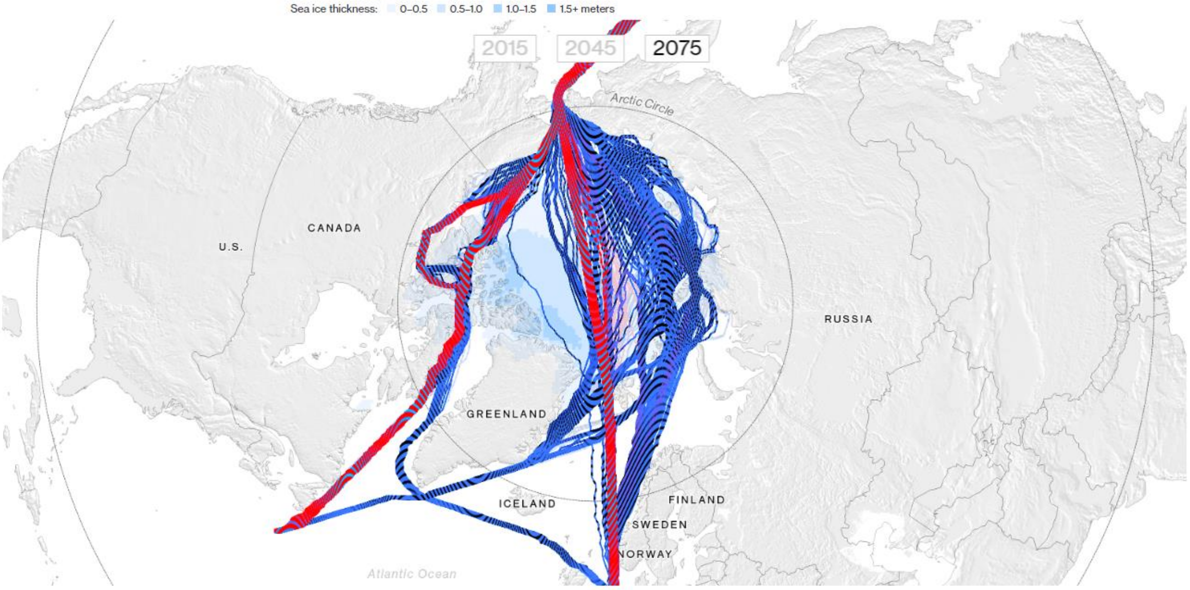

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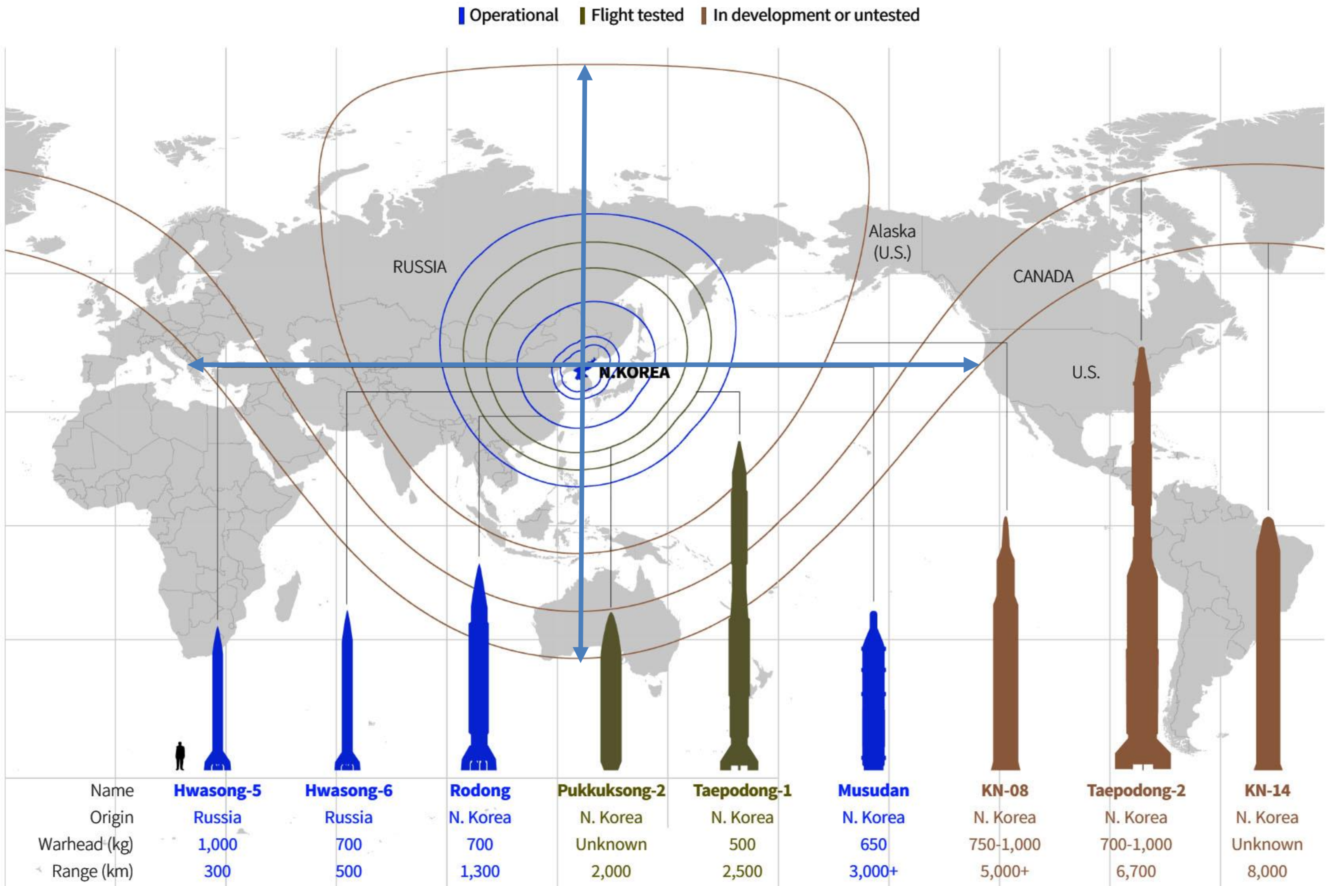
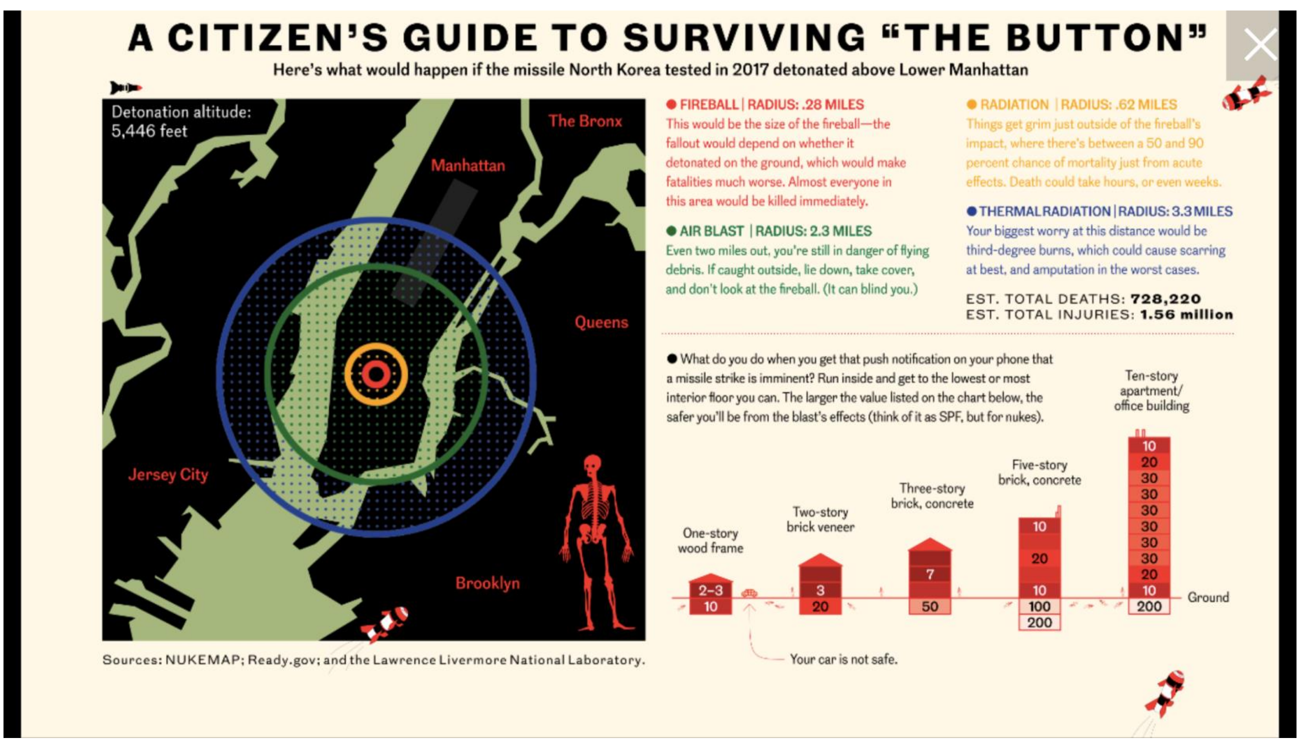
HW- Circles



1 The distance capability of the KN-14 missile is 8 thousand km from North Korea which is the center. Give me the circle of this analysis on Mathematica and give me the standard form of the equation



2- The detonation point is lower Manhattan Washington square park with the coordinates (40.73, -73.99) with an r^2 = 9 miles

Exercises

3-

Find the standard form of the equation for the circle with radius 3 and center $(-2, 7)$.

//4-

Sketch the graph of the circle defined by

$$(x-2)^2+(y+3)^2=4.$$

//5-

Find the standard form of the equation for the circle with radius 3 and center $(3, 1)$

//6-

Consider the equation $x^2+y^2-8y+7=0$, Find the center (h, k) ,and radius, r of this circle and graph the circle.

//7-

Consider the equation $x^2+y^2-14x+10y+38=0$, Find the center (h, k) ,and radius, r of this circle and graph the circle.

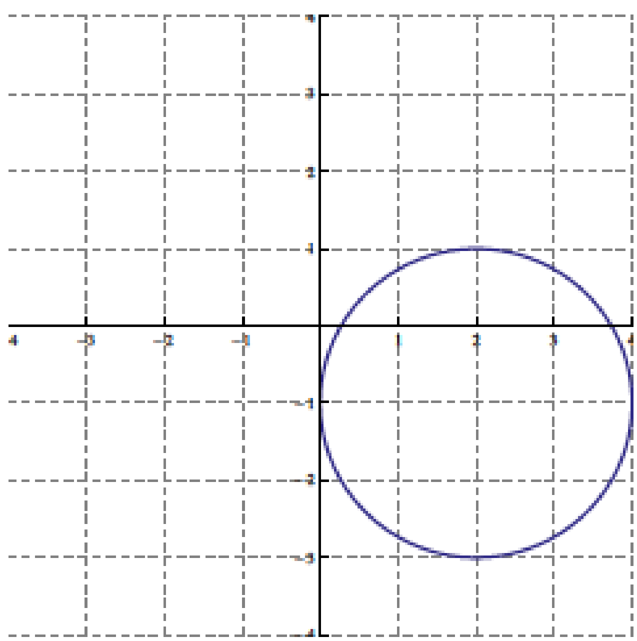
//8-

Consider the circle pictured below.

(a) Find the center (h, k) ,and radius, r of this circle.

(b) Write the equation of the circle in standard form.

(c) Find the intercepts.



// 9- only responsible in math 119 for the graph in mathematica.

A circle has the equation $7x^2 + 7y^2 - 42x - 28y - 84 = 0$. Graph the circle using the center (h,k) and radius r. Find the intercepts, if any, of the graph.

Use the graphing tool to graph the circle.

Click to enlarge

At what points do the x-intercepts occur? Select the correct choice below and fill in any answer boxes within your choice.

A. The x-intercept(s) is/are ☐.
(Type an ordered pair. Use a comma to separate answers as needed. Type exact answers for each coordinate, using radicals as needed.)

B. There is no x-intercept.

At what points do the y-intercepts occur? Select the correct choice below and fill in any answer boxes within your choice.

A. The y-intercept(s) is/are ☐.
(Type an ordered pair. Use a comma to separate answers as needed. Type exact answers for each coordinate, using radicals as needed.)

B. There is no y-intercept.

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For graphs to be enabled is Mathematica the following code is required.

```
Graphics [ Circle [ { h , k } ,  r ] , Axes - > True ]
```