Objective (07):
Describe functions, their domains, ranges, intervals on which they are increasing / decreasing / constant and be able to recognize and graph function of various types on the Cartesian plane.

Describe functions,

Function

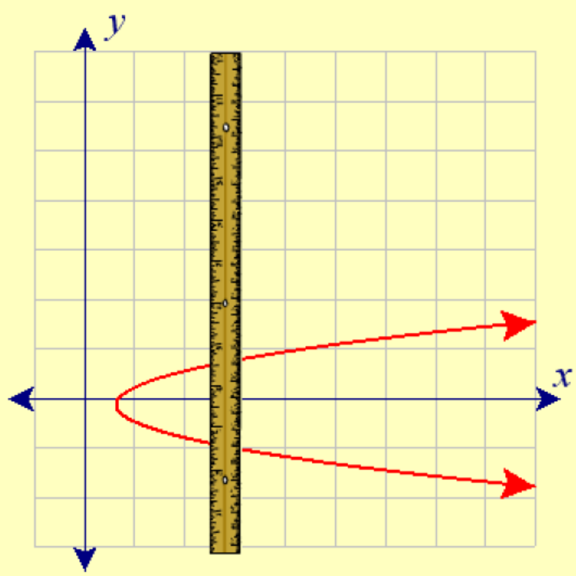
A **function** is a relation in which every element of the domain is paired with *exactly one* element of the range. Equivalently, a function is a relation in which no two distinct ordered pairs have the same first coordinate.

The Vertical Line Test

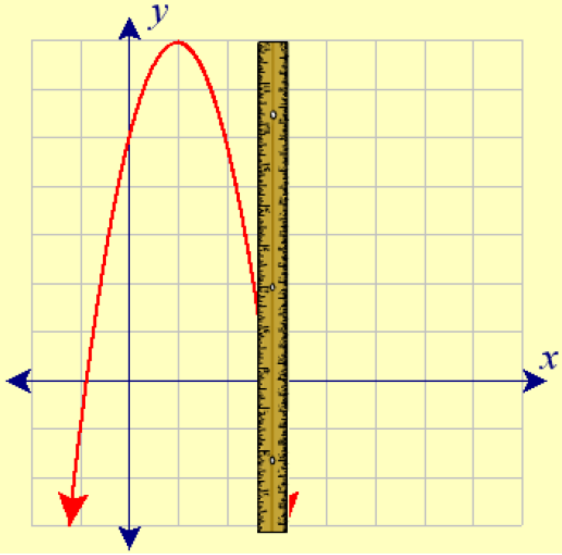
If a relation can be graphed in the Cartesian plane, the relation is a function if and only if no vertical line passes through the graph more than once. If even *one* vertical line intersects the graph of the relation two or more times, the relation fails to be a function.

The set $R = \{ (3, -2), (-1, 5), (-6, -4), (0, 0), (\sqrt{3}, -\pi) \}$ is a relation consisting of five ordered pairs. The domain of R is the set $\{ 3, -1, -6, 0, \sqrt{3} \}$, as these five numbers appear as first coordinates in the relation. The range of R is the set $\{ -2, 5, -4, 0, -\pi \}$, as these are the numbers that appear as second coordinates. The *graph* of this relation is simply a picture of the five ordered pairs plotted in the Cartesian plane, as shown below.

The relation graphed below is not a function, as there are many vertical lines that intersect the graph more than once. The ruler is one such vertical line.



The relation graphed below is a function. In this case, every vertical line in the plane intersects the graph exactly once.



their domains, ranges,

Relations, Domains, and Ranges

A **relation** is a set of ordered pairs. Any set of ordered pairs automatically relates the set of first coordinates to the set of second coordinates, and these sets have special names. The **domain** of a relation is the set of all the first coordinates, and the **range** of a relation is the set of all second coordinates.

[Linear and Quadratic Functions]

Linear

Linear functions ||1st degree with one variable. Y=ax+b a≠0 || f(x)=ax+b a≠0

Linear Functions

A **linear function** f of one variable, say the variable x, is any function that can be written in the form $f(x) = ax + b$, where a and b are real numbers. If $a \neq 0$, a function $f(x) = ax + b$ is also called a **first-degree function**.

Show examples:

linear equation (first degree equations)

f(x) = -4x+2 || show the graph and talk about domain and range, do a vertical line test.

Slope is m= -4/1 and y intercept is (0,2) do graph

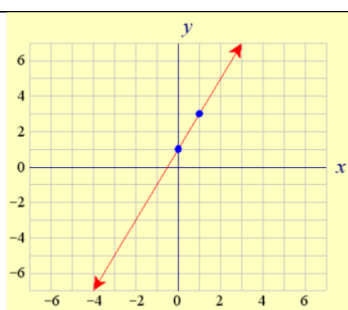
f(x) = (3+6x) / 3 || show graph and do domain , range, vertical line test.

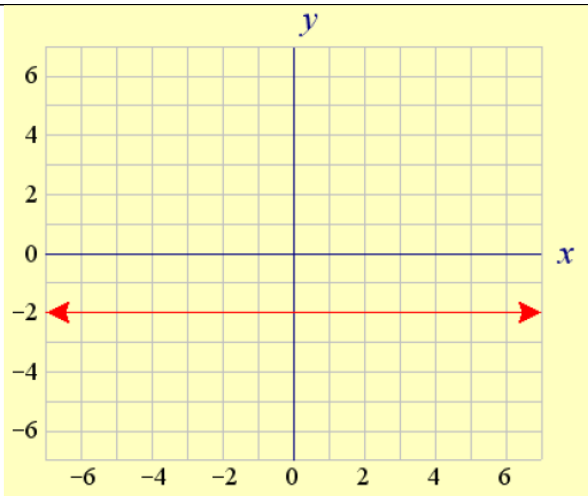
The function can be rewritten in the form of 1+ 2x by using the least common multiple 3/3 and the problem can rewritten as y = 2x+1 whereby the slope is m= 2/1 and y intercept is (0,1)

The domain will be all values for x for each and every point on this line because if a vertical line test is done it only hits the line once per point.

The range will be all values for y for each and every point on this line because if a vertical line test is done it only hits the line once per point.

Graph the following linear functions.

a. $f(x) = \dfrac{3+6x}{3}$	The function f can be rewritten as $f(x) = 2x + 1$, and in this form we recognize it as a line with a slope of 2 and a y-intercept of 1. To graph this function, then, we can plot the y-intercept $(0, 1)$ and locate another point on the line by moving up 2 and over to the right 1 unit, giving us the ordered pair $(1, 3)$. Once these two points have been plotted, connecting them with a straight line completes the process.
	

$g(x) = -2$	The graph of the function *g* is a straight line with a slope of 0 and a *y*-intercept of –2. A linear function with a slope of 0 is also called a constant function, as it turns any input into one fixed constant, in this case the number –2. The graph of a constant function is always a horizontal line.
 	

Quadratic

Quadratic functions ||2nd degree with two variables. Y= ax^2+bx+c a≠0 || f(x)=ax^2+bx+c a≠0

Show examples from the chap 2 quadratic equation

Quadratic Functions

A **quadratic, or second-degree, function** f of one variable, say the variable x, is any function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

x	$f(x)$
–3	9
–1	1
0	0
2	4

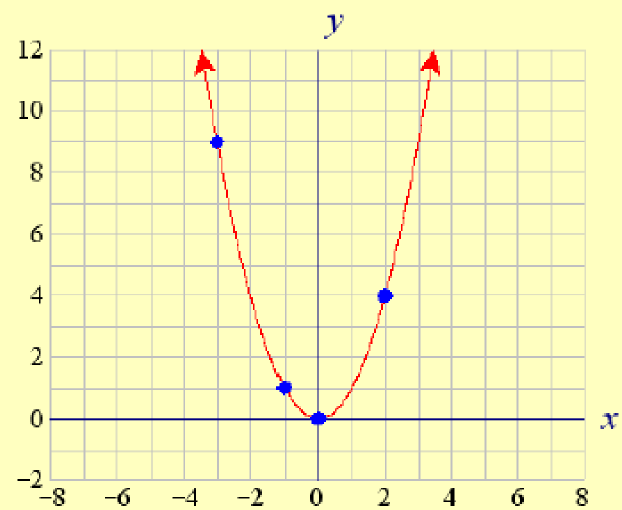
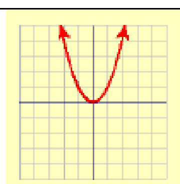
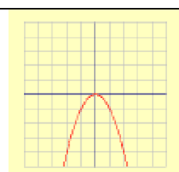


Figure 2: Graph of $f(x) = x^2$

f(x)= x^2		f(x) = -x^2		Talk about parabola, symmetric over axis.
Vertex (0,0) – in a positive parabola it is the lowest point and		Vertex (0,0) - the highest point if it is a negative parabola.		

Later we will pick up on the transformations of quadratic functions.

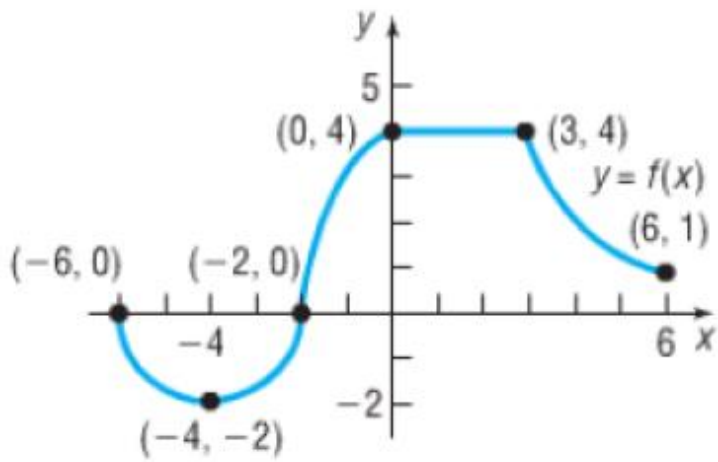
intervals

increasing / decreasing constant

3 Use a Graph to Determine Where a Function Is Increasing, Decreasing, or Constant

Consider the graph given in Figure 18. If you look from left to right along the graph of the function, you will notice that parts of the graph are going up, parts are going down, and parts are horizontal. In such cases, the function is described as *increasing, decreasing,* or *constant,* respectively.

Figure 18

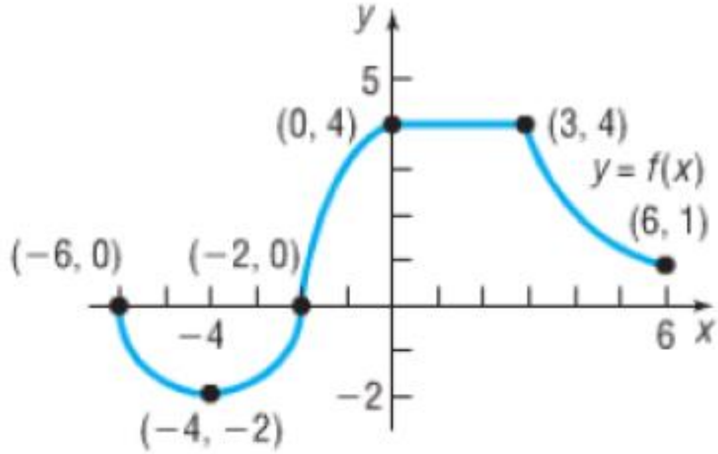


Determining Where a Function Is Increasing, Decreasing, or Constant from Its Graph

Where is the function in Figure 18 increasing? Where is it decreasing? Where is it constant?

Solution WARNING We describe the behavior of a graph in terms of its x-values. Do not say the graph in Figure 18 is increasing from the point (−4, 2) to the point (0, 4). Rather, say it is increasing on the interval (−4, 0). ■	For the following values of x, The graph is increasing from (-4,0) However these would be open intervals In the graph above the intervals are close aka solid Thus the interval notation of the above example would be [-4,0]

Figure 18



To answer the question of where a function is increasing, where it is decreasing, and where it is constant, we use strict inequalities involving the independent variable x, or we use open intervals* of x-coordinates. The function whose graph is given in Figure 18 is increasing on the open interval $(-4, 0)$ or for $-4 < x < 0$. The function is decreasing on the open intervals $(-6, -4)$ and $(3, 6)$ or for $-6 < x < -4$ and $3 < x < 6$. The function is constant on the open interval $(0, 3)$ or for $0 < x < 3$.

DEFINITIONS

A function f is **increasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

A function f is **decreasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.

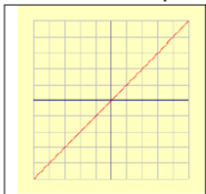
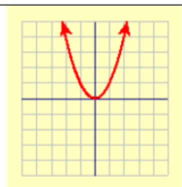
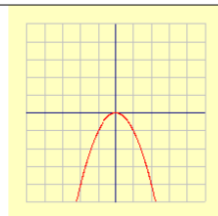
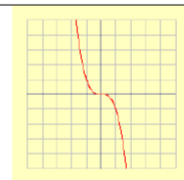
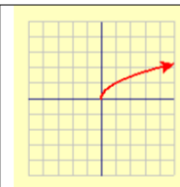
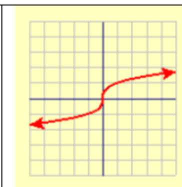
A function f is **constant** on an open interval I if, for all choices of x in I, the values $f(x)$ are equal.

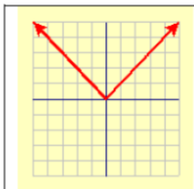
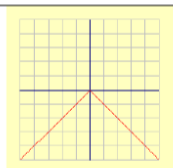
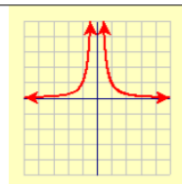
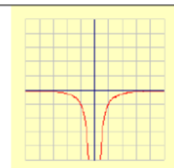
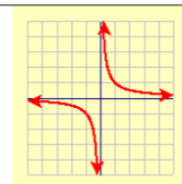
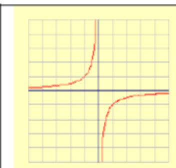
Decreasing open intervals	**Increasing open intervals**	**Constant open intervals**
(-6,-4) = (-6, 0) → (-4,0) = -6 < x < -4 (3, 6) = (3, 0) → (6, 0) = 3 < x < 6	(-4, 0) = (-4, 0) → (0, 0) = -4 < x < 0	(0, 3) = (0, 0) → (3, 0) = 0 < x < 3

Decreasing closed intervals	**Increasing closed intervals**	**Constant closed intervals**

recognize functions

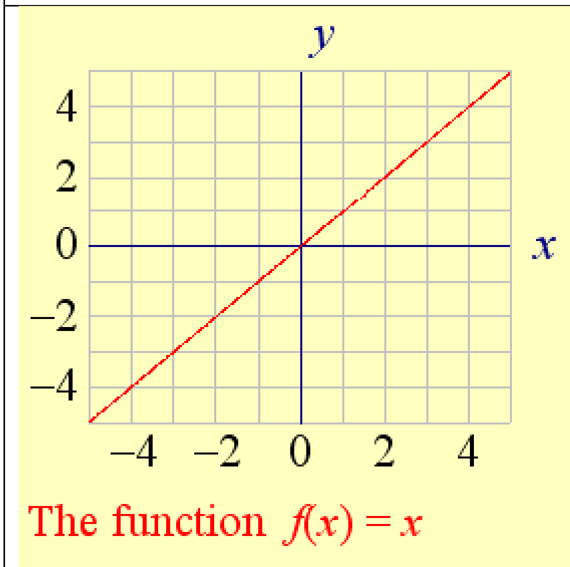
Quadratic Equations and other common functions

						
f(x)= x or x^1 positive odd	f(x)= x^2 positive even	f(x)= -x^2 negative even	f(x)= x^3 odd positive	f(x)= -x^3 odd negative	f(x)= x^1/2 even	f(x)= x^1/3 odd

						
f(x)= |x|	f(x)= -2 |x|	f(x)= 1/x^2 positive even	f(x)= - (1/x^2) negative even	f(x)= 1/x^1 odd positive	f(x)= - (1/x^1) negative odd	f(x)=

graph functions of various types on the Cartesian plane.

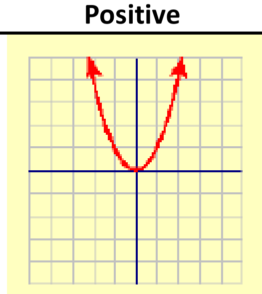
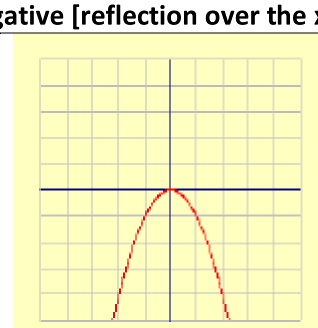
Identity line aka linear line aka linear function

Positive	Negative
	

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	

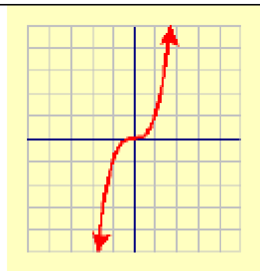
Parabola

Positive	Negative [reflection over the x-axis]
 f(x)= x^2 positive even	 f(x)= -x^2 negative even

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Transformations	a(x-h)^2 + k

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Tranformations	-a(x-h)^2 + k

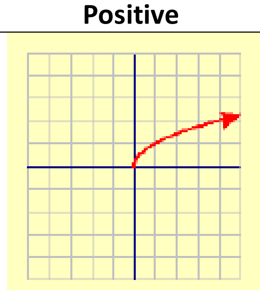
Cubic function

Positive	**Negative [reflection over the x-axis]**
 f(x)= x^3 odd positive	 f(x)= -x^3 odd negative

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Tranformations	-a(x-h)^3 + k

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Tranformations	-a(x-h)^3 + k

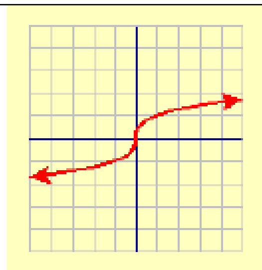
Square root

Positive	**Negative [reflection over the x-axis]**
 f(x)= x^1/2 even	

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Tranformations	a$\sqrt[2]{x-h}$ +k

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Tranformations	-a$\sqrt[2]{x-h}$ +k

Cubic root

Positive	Negative [reflection over the x-axis]
 f(x)= x^1/3 odd	

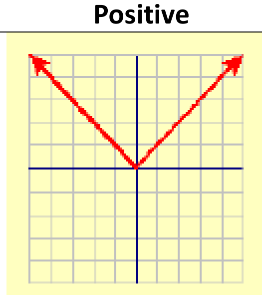
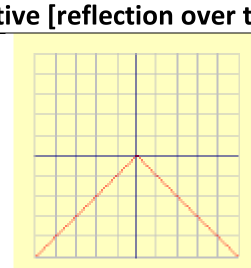
Positive

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Transformations	a$\sqrt[3]{x-h}$ +k

Negative [reflection over the x-axis]

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Transformations	-a$\sqrt[3]{x-h}$ +k

Absolute Value

Positive	Negative [reflection over the x-axis]
 f(x)= |x|	 f(x)= -2 |x|

Positive

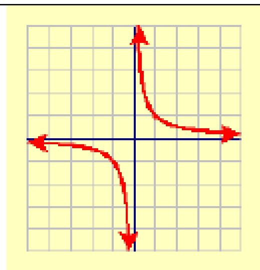
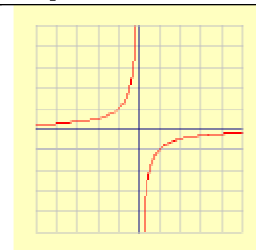
Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Transformations	a|x-h| + k

Negative [reflection over the x-axis]

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Transformations	-a|x-h| + k

Asymptotes

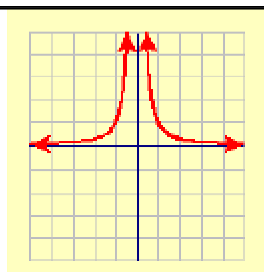
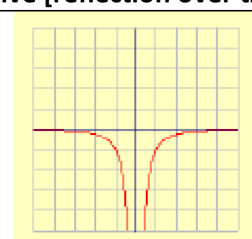
Odd

Positive	Negative [reflection over the x-axis]
 f(x)= 1/x^1 odd positive	 f(x)= - (1/x^1) negative odd

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Transformations	$\frac{1}{(x-h)^1}+k$

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Transformations	$-\frac{1}{(x-h)^1}+k$

Even

Positive	Negative [reflection over the x-axis]
 f(x)= 1/x^2 positive even	 f(x)= - (1/x^2) negative even

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Transformations	$\frac{1}{(x-h)^2}+k$

Domain	
Range	
Vertex	
Symmetry	
Axis of Symmetry	
Transformations	$-\frac{1}{(x-h)^2}+k$

Transformations of quadratic functions

Quadratic

Quadratic functions ||2^{nd} degree with two variables. Y= ax^2+bx+c a≠0 || f(x)=ax^2+bx+c a≠0

Show examples from the chap 2 quadratic equation

Quadratic Functions

A **quadratic, or second-degree, function** f of one variable, say the variable x, is any function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

x	$f(x)$
−3	9
−1	1
0	0
2	4

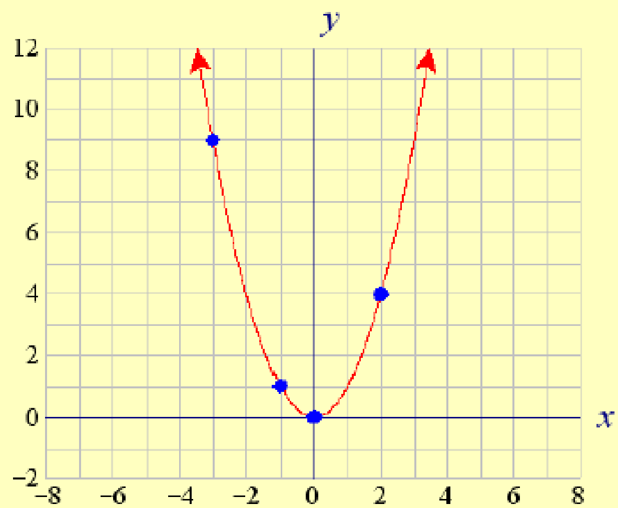
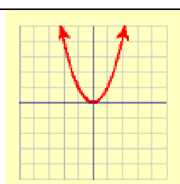
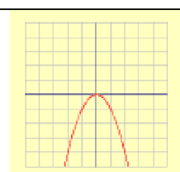
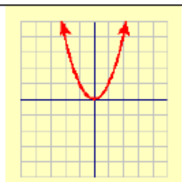
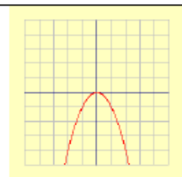
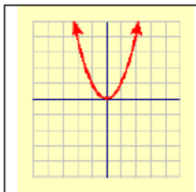
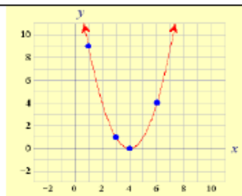


Figure 2: Graph of $f(x) = x^2$

f(x)= x^2		f(x) = -x^2		Talk about parabola, symmetric over axis.
Vertex (0,0) – in a positive parabola it is the lowest point and			Vertex (0,0) - the highest point if it is a negative parabola.	

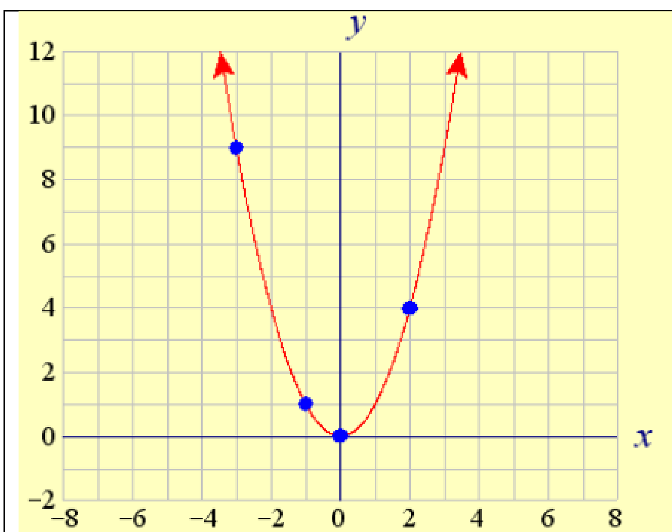
f(x)= x^2		f(x) = -x^2		Talk about parabola, symmetric over axis.
Vertex (0,0) – in a positive parabola it is the lowest point and			Vertex (0,0) - the highest point if it is a negative parabola.	

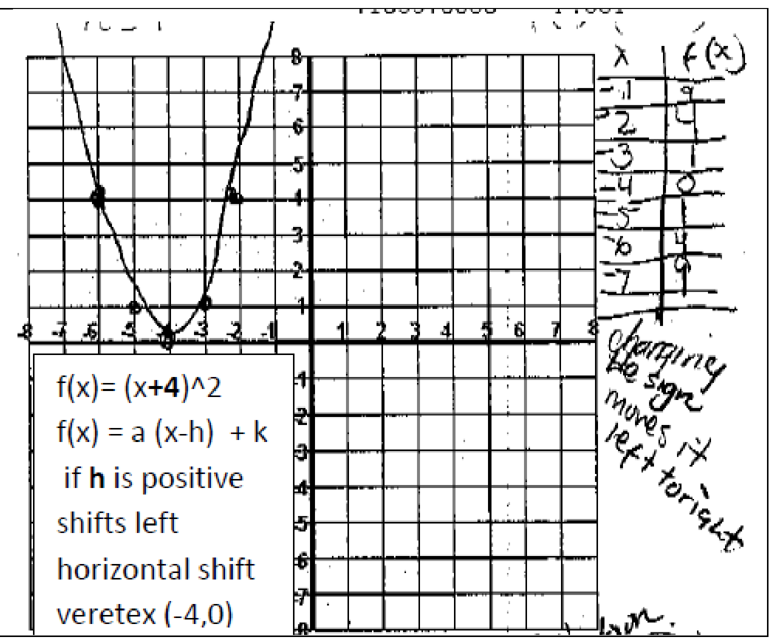
 f(x)= x^2	 f(x)= (x-4)^2	<table><tr><th>x</th><th>g(x)</th></tr><tr><td>1</td><td>9</td></tr><tr><td>3</td><td>1</td></tr><tr><td>4</td><td>0</td></tr><tr><td>6</td><td>4</td></tr></table> Vertex (4, 0)	**Vertex form of a Quadratic Function** f(x) = a (x-h)^2 +k **a** | | if a > 1 = narrower or skinnier | | **a** | |if 0<a<1 = broader or fatter **h** | | shifts the function (+) left or (-) right depending on the sign. **k** | | shifts the function up or down depending on the sign.
Vertex (0,0)	Vertex (4,0)		***DO the f(x)= (x-4)^2 from the graph paper done.***

In order to shift the parabola horizontally on the x-axis
we add a positive value to the existing term to move it to the **left**.
|| f (x) = (x+4)^2 || thus changing the vertex from (0,0) to (-4,0)

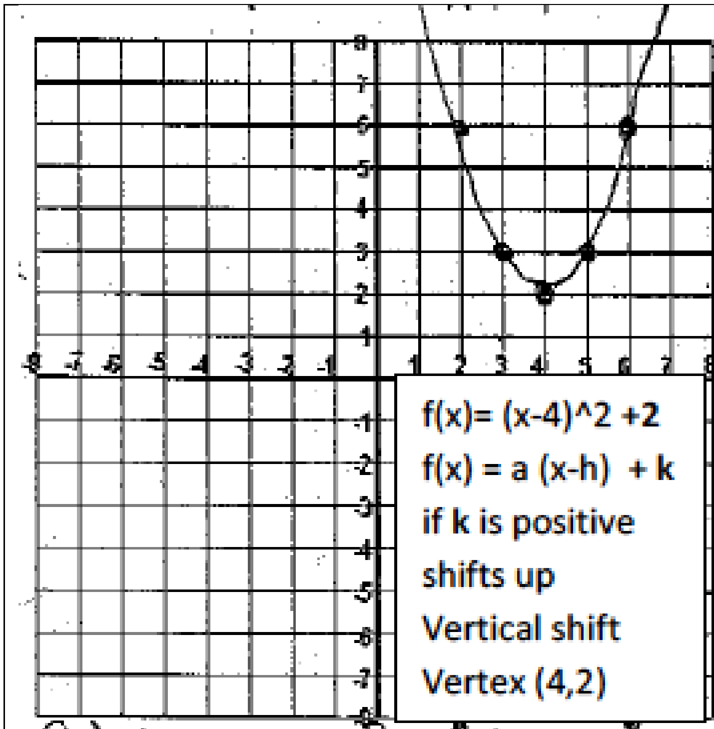
we add a negative value to the existing term to move it to the **right**.
|| f (x) = (x-4)^2 || thus changing the vertex from (0,0) to (4,0)

F(x)= a(x-h)^2 + k	H = horizontal shift. Always doe the opposite.
	K= vertical shift
	a=skinny a>1 or wide <0a<1

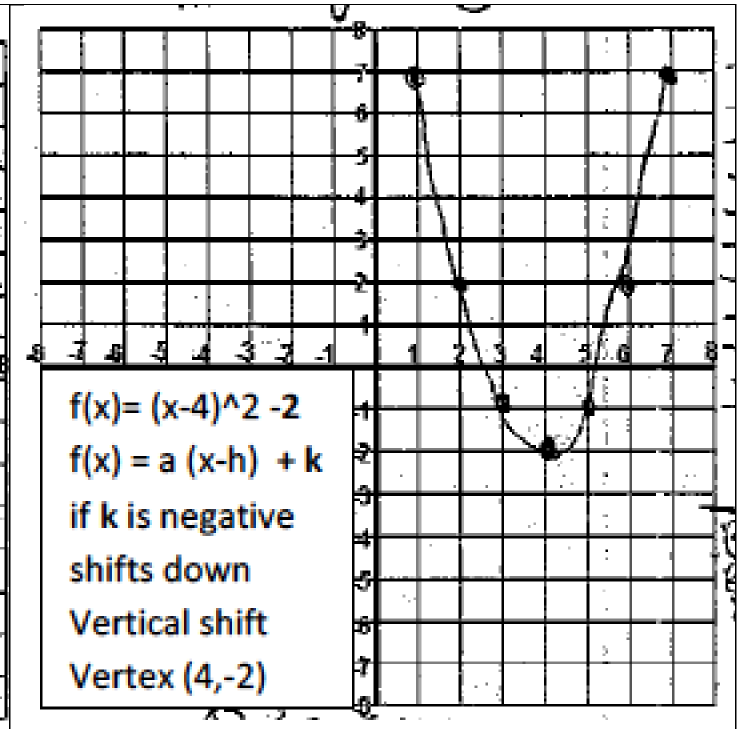




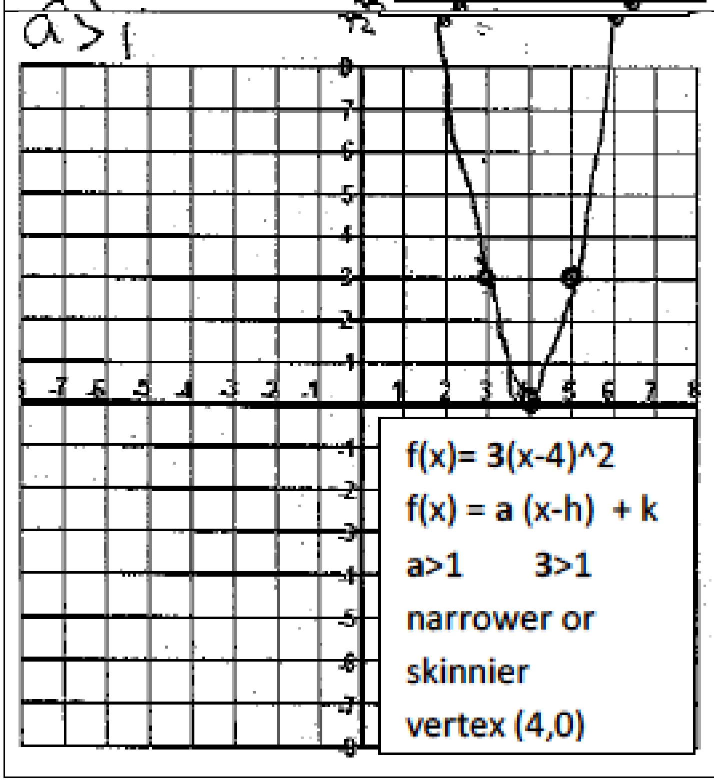
f(x)= (x+**4**)^2
f(x) = a (x-h) + k
if **h** is positive
shifts left
horizontal shift
veretex (-4,0)



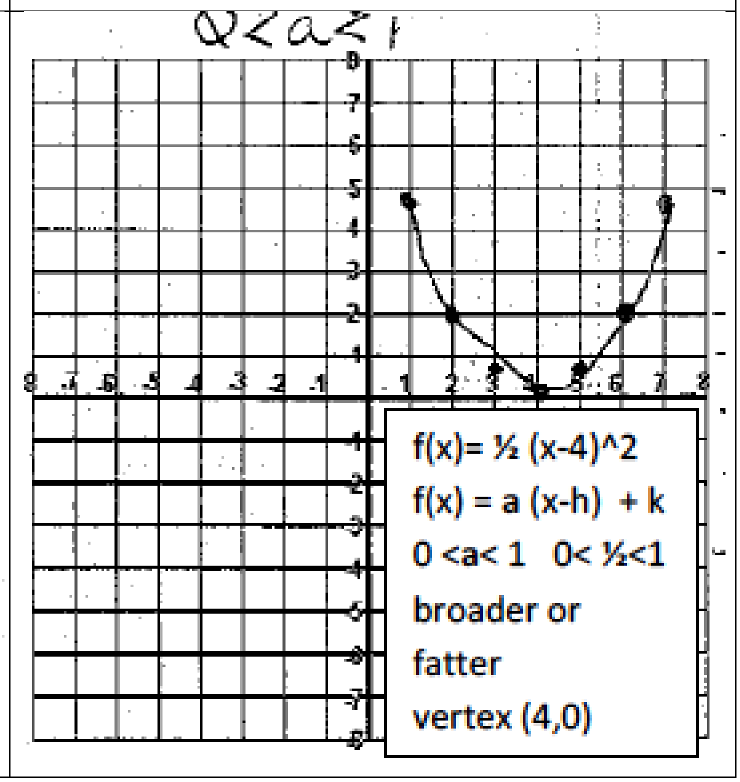
f(x)= (x-4)^2 +2
f(x) = a (x-h) + k
if k is positive
shifts up
Vertical shift
Vertex (4,2)



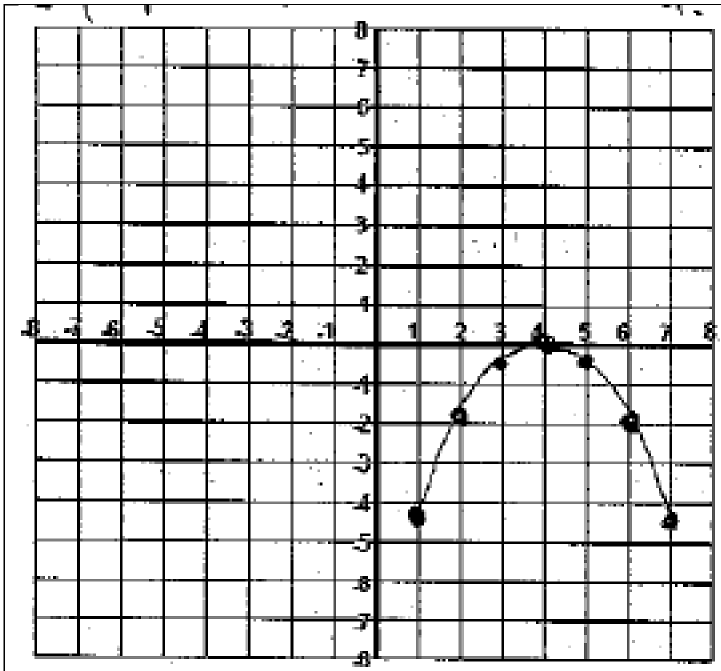
f(x)= (x-4)^2 -2
f(x) = a (x-h) + k
if k is negative
shifts down
Vertical shift
Vertex (4,-2)

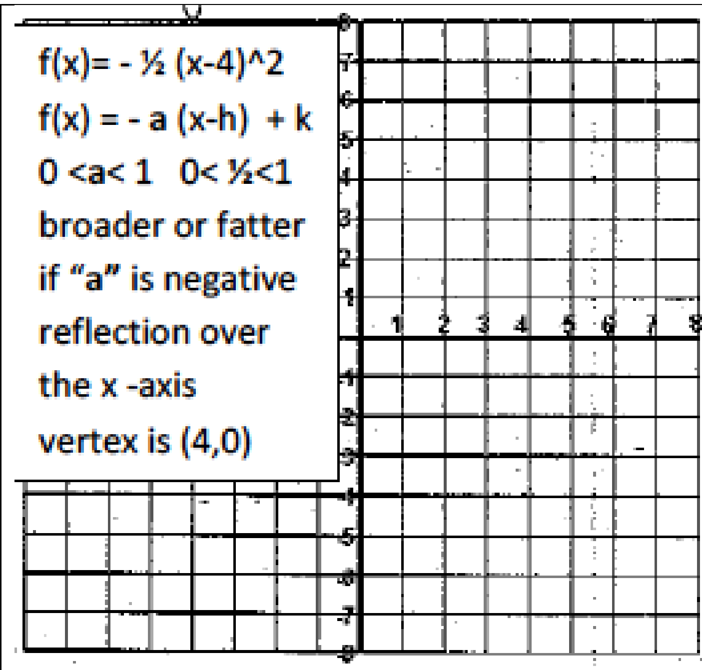


f(x)= 3(x-4)^2
f(x) = a (x-h) + k
a>1   3>1
narrower or
skinnier
vertex (4,0)



f(x)= ½ (x-4)^2
f(x) = a (x-h) + k
0 <a< 1   0< ½<1
broader or
fatter
vertex (4,0)




f(x)= - ½ (x-4)^2
f(x) = - a (x-h) + k
0 <a< 1 0< ½<1
broader or fatter
if "a" is negative
reflection over
the x -axis
vertex is (4,0)

Consider the following quadratic function.

$$t(x) = -(x+5)^2 - 3$$

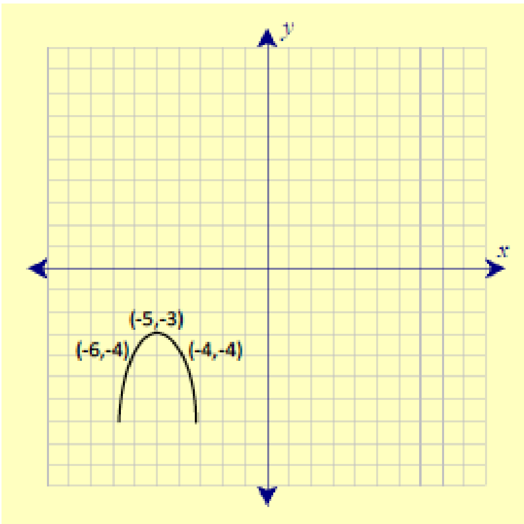
Step 1. Find the vertex of this function.

Step 1: (−5, −3)

Step 2. Determine the number of x-intercept(s), then enter the x-intercept(s), if any, of this function as ordered pair(s) below.

Step 2: None, the function does not touch the x-axis.

Step 3. Graph this quadratic function by identifying two points on the parabola other than the vertex and the x-intercepts.



A: **B:**

Points
(-6,-4)
(-4,-4)
(-7,-7)
(-3,-7)

Step 3: A= (−6, −4) , B= (−4, −4)

x	f(x) = y	$-(x+5)^2 -3$	PLOT
-7	-7	-((-7)+5)^2 -3 = -(-2)^2 – 3 = -4-3 = -7	(-7-7)
-6	-4		(-6,-4)
-5	-3		(-5,-3) vertex
-4	-4		(-4,-4)
-3	-7		(-3,-7)

See page 11 problem 33 of chapter 3 review. q(x) = -2x^2 +4x

(1)Find the vertex

(1,2) test using

a < 0 then f(-(b/2a)) is the max point // a > 0 then f(-(b/2a)) is the min point vertex (x,y) vertex ((-(b/2a)),f(-(b/2a)))

(2)Determine x intercepts

x	f(x) =	-2x^2 +4x	PLOT
-1	-6		(-,5)
0	0	-2(0)^2+ 4(0)= 0	(0,0) x-intercept
1	2		(1,2) vertex
2	0		(2,0) x-intercept
3	-6		(1,5)

-2x^2 +4x // -2x (x-2) // -2x=0 thus x=0 // x-2 = 0 thus x=2 // x intercepts are (0,0) & (2,0)

(3)graph

Circles are not functions they do not pass the vertical line test we will learn more about them in the next presentation.