Properties of Logs

SUMMARY Properties of Logarithms

In the list that follows, a, b, M, N, and r are real numbers. Also, $a > 0, a \neq 1, b > 0, b \neq 1, M > 0$, and $N > 0$.

Definition	$y = \log_a x$ means $x = a^y$	
Properties of logarithms	$\log_a 1 = 0; \quad \log_a a = 1$	$\log_a M^r = r \log_a M$
	$a^{\log_a M} = M; \quad \log_a a^r = r$	$a^x = e^{x \ln a}$
	$\log_a(MN) = \log_a M + \log_a N$	If $M = N$, then $\log_a M = \log_a N$.
	$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$	If $\log_a M = \log_a N$, then $M = N$.
Change-of-Base Formula	$\log_a M = \frac{\log_b M}{\log_b a}$	

Properties of Logarithms (Recall that logs are only defined for positive values of x.)

For the natural logarithm

1. $\ln xy = \ln x + \ln y$
2. $\ln \frac{x}{y} = \ln x - \ln y$
3. $\ln x^y = y \cdot \ln x$
4. $\ln e^x = x$
5. $e^{\ln x} = x$

For logarithms base a

1. $\log_a xy = \log_a x + \log_a y$
2. $\log_a \frac{x}{y} = \log_a x - \log_a y$
3. $\log_a x^y = y \cdot \log_a x$
4. $\log_a a^x = x$
5. $a^{\log_a x} = x$

General Properties

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = 0$
4. $b^{\log_b x} = x$

Useful Identities for Logarithms

For the natural logarithm

1. $\ln e = 1$
2. $\ln 1 = 0$

For logarithms base a

1. $\log_a a = 1$, for all $a > 0$
2. $\log_a 1 = 0$, for all $a > 0$

Natural Logarithms

1. $\ln 1 = 0$
2. $\ln e = 1$
3. $\ln e^x = x$
4. $e^{\ln x} = x$

Properties of Natural Logarithms

1. $\ln 1 = 0$ since $e^0 = 1$.
2. $\ln e = 1$ since $e^1 = e$.
3. $\ln e^x = x$ and $e^{\ln x} = x$ — inverse property
4. If $\ln x = \ln y$, then $x = y$. — one-to-one property

Exponential Laws

$a^0 = 1$, for $a \neq 0$

- $x^1 = x$
- $x^0 = 1$
- $x^{-1} = 1/x$

$\log_a 1 = 0 \qquad \log_a a = 1$

Properties of Logarithms

In the following properties, M, N, and a are positive real numbers, $a \neq 1$, and r is any real number.

The Log of a Product Equals the Sum of the Logs

$$\log_a(MN) = \log_a M + \log_a N \qquad (3)$$

The Log of a Quotient Equals the Difference of the Logs

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N \qquad (4)$$

The Log of a Power Equals the Product of the Power and the Log

$$\log_a M^r = r \log_a M \qquad (5)$$

$$a^x = e^{x \ln a} \qquad (6)$$

Exercises

Use the properties of logarithms to expand or simplify the following expression as much as possible. Simplify any numerical expressions that can be evaluated without a calculator.

$$\log_a 7 + 4\log_a 3 = \log_a 7 + \log_a 3^4 \qquad r\log_a M = \log_a M^r$$
$$= \log_a 7 + \log_a 81$$
$$= \log_a(7 \cdot 81) \qquad \log_a M + \log_a N = \log_a(M \cdot N)$$
$$= \log_a 567$$

Product Rule

$$\log_a x + \log_a 9 + \log_a(x^2+1) - \log_a 5 = \log_a(9x) + \log_a(x^2+1) - \log_a 5$$
$$= \log_a[9x(x^2+1)] - \log_a 5$$
$$= \log_a\left[\frac{9x(x^2+1)}{5}\right]$$

Quotient Rule