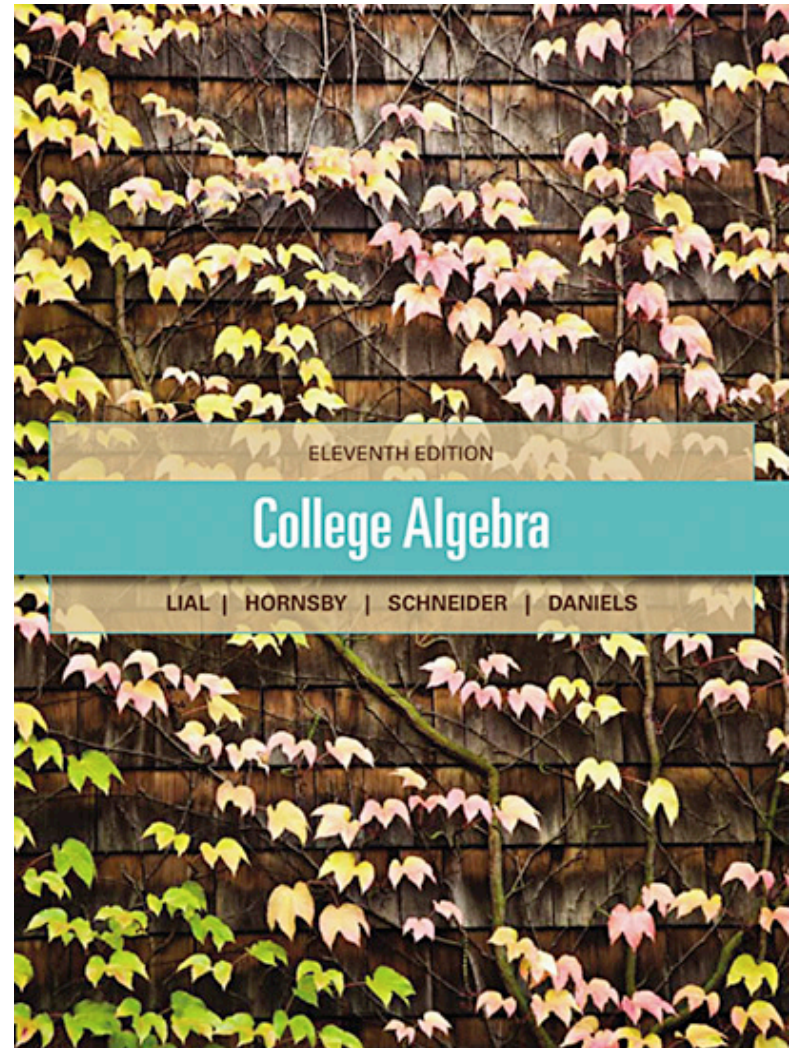


R

Review of Basic Concepts

Sections R.5–R.7



R

Review of Basic Concepts

R.5 Rational Expressions

R.6 Rational Exponents

R.7 Radical Expressions

R.5 Rational Expressions

Rational Expressions ▪ Lowest Terms of a Rational Expression ▪
Multiplication and Division ▪ Addition and Subtraction ▪
Complex Fractions

R.5 Example 1 Finding the Domain (page 41)

Find the domain of the rational expression.

$$\frac{(x - 7)(x - 1)}{(x + 3)(x - 1)}$$

Set the denominator equal to zero.

$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -3 \quad \text{or} \quad x = 1$$

$$\{x \mid x \neq -3, 1\}$$

R.5 Example 2(a) Writing Rational Expressions in Lowest Terms (page 42)

Write the rational expression in lowest terms.

$$\begin{aligned} \text{(a)} \quad \frac{12x^2 - 30x}{4x^2 - 25} &= \frac{6x(2x - 5)}{(3x + 5)(2x - 5)} && \text{Factor.} \\ &= \frac{6x}{2x + 5} && \text{Divide out the common factor.} \end{aligned}$$

R.5 Example 2(b) Writing Rational Expressions in Lowest Terms (page 42)

Write the rational expression in lowest terms.

$$\begin{aligned} \text{(b)} \quad \frac{x^2 - 8x - 16}{8x - 2x^2} &= \frac{(x - 4)^2}{2x(4 - x)} && \text{Factor.} \\ &= -\frac{(x - 4)^2}{2x(x - 4)} && \text{Multiply numerator and denominator by } -1. \\ &= -\frac{x - 4}{2x} && \text{Divide out the common factor.} \\ &= \frac{-(x - 4)}{2x} \text{ or } \frac{4 - x}{2x} \end{aligned}$$

R.5 Example 3(a) Multiplying or Dividing Rational Expressions (page 43)

Multiply.

$$\begin{aligned}\frac{6z^6}{7} \cdot \frac{28}{9z^2} &= \frac{6z^6 \cdot 28}{7 \cdot 9z^2} && \text{Multiply.} \\ &= \frac{2 \cdot 3 \cdot 4 \cdot 7 \cdot z^2 \cdot z^4}{7 \cdot 3 \cdot 3 \cdot z^2} && \text{Factor.} \\ &= \frac{8z^4}{3} && \text{Divide out common factors,} \\ &&& \text{then simplify.}\end{aligned}$$

R.5 Example 3(b) Multiplying or Dividing

Rational Expressions (page 43)

Multiply.

$$\frac{4n^2 + 3n - 10}{2n^2 + 3n - 2} \cdot \frac{2n - 1}{n + 4}$$

$$= \frac{(n + 2)(4n - 5)}{(n + 2)(2n - 1)} \cdot \frac{(2n - 1)}{(n + 4)}$$

Factor.

$$= \frac{(n + 2)(4n - 5)(2n - 1)}{(n + 2)(2n - 1)(n + 4)}$$

Multiply.

$$= \frac{4n - 5}{n + 4}$$

Divide out common factors,
then simplify.

R.5 Example 3(c) Multiplying or Dividing Rational Expressions (page 43)

Divide.

$$\frac{5z^2 - 16z + 3}{z^2 + z - 12} \div \frac{30z^2 - 6z}{2z^3 + 8z^2}$$

$$= \frac{5z^2 - 16z + 3}{z^2 + z - 12} \cdot \frac{2z^3 + 8z^2}{30z^2 - 6z}$$

Multiply by the reciprocal of the divisor.

$$= \frac{(5z - 1)(z - 3)}{(z + 4)(z - 3)} \cdot \frac{2z \cdot z(z + 4)}{3 \cdot 2z(5z - 1)}$$

Factor.

$$= \frac{z}{3}$$

Multiply, then divide out common factors.

R.5 Example 3(d) Multiplying or Dividing

Rational Expressions (page 43)

Multiply.

$$\frac{x^2 - 1}{x^3 - 1} \cdot \frac{xy - 2y + 3x - 6}{xy + 3x + y + 3}$$

$$= \frac{(x - 1)(x + 1)}{(x - 1)(x^2 + x + 1)} \cdot \frac{y(x - 2) + 3(x - 2)}{x(y + 3) + (y + 3)}$$

Factor.

$$= \frac{(x - 1)(x + 1)}{(x - 1)(x^2 + x + 1)} \cdot \frac{(y + 3)(x - 2)}{(x + 1)(y + 3)}$$

$$= \frac{x - 2}{x^2 + x + 1}$$

Multiply, then divide out common factors.

R.5 Example 4(a) Adding or Subtracting Rational Expressions (page 44)

Add $\frac{3}{10z^4} + \frac{2}{15z^2}$

Find the LCD:

$$\left. \begin{array}{l} 10z^4 = 2 \cdot 5 \cdot z^4 \\ 15z^2 = 3 \cdot 5 \cdot z^2 \end{array} \right\} \text{LCD} = 2 \cdot 3 \cdot 5 \cdot z^4 = 30z^4$$

$$\begin{aligned} \frac{3}{10z^4} + \frac{2}{15z^2} &= \frac{3 \cdot 3}{10z^4 \cdot 3} + \frac{2 \cdot 2z^2}{15z^2 \cdot 2z^2} \\ &= \frac{9}{30z^4} + \frac{4z^2}{30z^4} = \frac{9 + 4z^2}{30z^4} \end{aligned}$$

R.5 Example 4(b) Adding or Subtracting Rational Expressions (page 44)

Add $\frac{7}{m-5} + \frac{2m}{5-m}$

Find the LCD: $\left. \begin{array}{l} m-5 = m-5 \\ 5-m = (-1)(m-5) \end{array} \right\} \text{LCD} = m-5$

$$\begin{aligned} \frac{7}{m-5} + \frac{2m}{5-m} &= \frac{7}{m-5} + \frac{2m(-1)}{(5-m)(-1)} \\ &= \frac{7}{m-5} + \frac{-2m}{m-5} \\ &= \frac{7-2m}{m-5} \quad \text{or} \quad \frac{2m-7}{5-m} \end{aligned}$$

R.5 Example 4(c) Adding or Subtracting

Rational Expressions (page 44)

$$\text{Subtract } \frac{4}{(x-3)(x+5)} - \frac{6}{(x+5)(x-5)}$$

Find the LCD: $(x-3)(x+5)(x-5)$

$$\begin{aligned} & \frac{4}{(x-3)(x+5)} - \frac{6}{(x+5)(x-5)} \\ &= \frac{4(x-5)}{(x-3)(x+5)(x-5)} - \frac{6(x-3)}{(x-3)(x+5)(x-5)} \\ &= \frac{4x-20-(6x-18)}{(x-3)(x+5)(x-5)} = \frac{-2x-2}{(x-3)(x+5)(x-5)} \end{aligned}$$

R.5 Example 5(a) Simplifying Complex Fractions (page 46)

Simplify $\frac{3 + \frac{4}{x^2}}{6 - \frac{1}{x^2}}$

Multiply the numerator and denominator by the LCD of all the fractions, x^2 .

$$\frac{3 + \frac{4}{x^2}}{6 - \frac{1}{x^2}} = \frac{x^2 \left(3 + \frac{4}{x^2} \right)}{x^2 \left(6 - \frac{1}{x^2} \right)} = \frac{3x^2 + 4}{6x^2 - 1}$$

R.5 Example 5(b) Simplifying Complex Fractions (page 46)

Simplify $\frac{\frac{1}{z+1} - \frac{1}{z-1}}{\frac{1}{z} + \frac{1}{z+1}}$

Multiply the numerator and denominator by the LCD of all the fractions, $z(z+1)(z-1)$.

$$\begin{aligned}\frac{\frac{1}{z+1} - \frac{1}{z-1}}{\frac{1}{z} + \frac{1}{z+1}} &= \frac{z(z+1)(z-1)\left(\frac{1}{z+1} - \frac{1}{z-1}\right)}{z(z+1)(z-1)\left(\frac{1}{z} + \frac{1}{z+1}\right)} \\ &= \frac{z(z-1) - z(z+1)}{(z+1)(z-1) + z(z-1)} \\ &= \frac{z^2 - z - z^2 - z}{z^2 - 1 + z^2 - z} = \frac{-2z}{2z^2 - z - 1}\end{aligned}$$

R.6 Rational Exponents

Negative Exponents and the Quotient Rule ▪
Rational Exponents ▪ Complex Fractions Revisited

R.6 Example 1 Using the Definition of a Negative Exponent (page 50)

Evaluate each expression.

$$(a) 10^{-3} \qquad (b) -5^{-1} \qquad (c) \left(\frac{4}{9}\right)^{-2}$$

$$(a) 10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$$

$$(b) -5^{-1} = -\frac{1}{5}$$

$$(c) \left(\frac{4}{9}\right)^{-2} = \frac{1}{\left(\frac{4}{9}\right)^2} = \frac{1}{\frac{16}{81}} = \frac{81}{16}$$

R.6 Example 1 Using the Definition of a Negative Exponent (cont.)

Write the expression without negative exponents.

$$(d) \quad mn^{-4} = \frac{m}{n^4}$$

$$(e) \quad (mn)^{-4} = \frac{1}{(mn)^4} = \frac{1}{m^4 n^4}$$

R.6 Example 2 Using the Quotient Rule (page 51)

Simplify each expression.

$$(a) \frac{15^8}{15^3} = 15^{8-3} = 15^5 \quad (b) \frac{y^4}{y^{-9}} = y^{4-(-9)} = y^{13}$$

$$(c) \frac{35r^6}{25r^{-4}} = \frac{7r^{6-(-4)}}{5} = \frac{7r^{10}}{5}$$

$$(d) \frac{34a^8b^{11}}{51a^{12}b^5} = \frac{34}{51} \cdot \frac{a^8}{a^{12}} \cdot \frac{b^{11}}{b^5} = \frac{2}{3} a^{8-12} b^{11-5}$$
$$= \frac{2}{3} a^{-4} b^6 = \frac{2b^6}{3a^4}$$

R.6 Example 3(a) Using Rules for Exponents (page 51)

Simplify.

$$\begin{aligned}5x^3 (2^{-1}x^4)^{-3} &= 5x^3 (2^{-1(-3)} x^{4(-3)}) \\ &= 5x^3 (2^3 x^{-12}) \\ &= 5x^{3-12} (8) \\ &= 5x^{-9} (8) \\ &= \frac{40}{x^9}\end{aligned}$$

R.6 Example 3(b) Using Rules for Exponents (page 51)

Simplify.

$$\begin{aligned}\frac{30r^4s^{-9}}{45r^{-6}s^3} &= \frac{30}{45} \cdot \frac{r^4}{r^{-6}} \cdot \frac{s^{-9}}{s^3} \\ &= \frac{2}{3} \cdot r^{4-(-6)} \cdot s^{-9-3} \\ &= \frac{2}{3} \cdot r^{10} \cdot s^{-12} \\ &= \frac{2r^{10}}{3s^{12}}\end{aligned}$$

R.6 Example 3(c) Using Rules for Exponents (page 51)

Simplify.

$$\begin{aligned}\frac{(4b^3)^{-2} (4b^{-1})^{-3}}{(4^{-1}b^3)^{-4}} &= \frac{4^{-2} b^{3(-2)} (4^{-3}) b^{-1(-3)}}{4^{-1(-4)} b^{3(-4)}} \\ &= \frac{4^{-2} \cdot 4^{-3} b^{-6} b^3}{4^4 b^{-12}} \\ &= \frac{4^{-2-3} b^{-6+3}}{4^4 b^{-12}} = \frac{4^{-5} b^{-3}}{4^4 b^{-12}} \\ &= 4^{-5-4} b^{-3-(-12)} = 4^{-9} b^9 = \frac{b^9}{4^9}\end{aligned}$$

R.6 Example 4 Using the Definition of $a^{1/n}$ (page 52)

Evaluate each expression.

$$(a) 49^{1/2} = 7$$

$$(b) -144^{1/2} = -12$$

$$(c) -(144)^{1/2} = -12$$

$$(d) 64^{1/6} = 2$$

$$(e) (-64)^{1/6}$$

not a real number

$$(f) -64^{1/6} = -2$$

$$(g) (-125)^{1/3} = -5$$

$$(h) -64^{1/3} = -4$$

R.6 Example 5 Using the Definition of $a^{m/n}$ (page 53)

Evaluate each expression.

$$(a) \quad 81^{3/4} = \left(81^{1/4}\right)^3 = 3^3 = 27$$

$$(b) \quad 25^{3/2} = \left(25^{1/2}\right)^3 = 5^3 = 125$$

$$(c) \quad -4^{5/2} = -\left(4^{1/2}\right)^5 = -2^5 = -32$$

R.6 Example 5 Using the Definition of $a^{m/n}$ (cont.)

Evaluate each expression.

$$(d) (-64)^{2/3} = \left[(-64)^{1/3} \right]^2 = (-4)^2 = 16$$

$$(e) 216^{-2/3} = \left(216^{1/3} \right)^{-2} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$$

(f) $(-100)^{3/2}$ is not a real number because $(-100)^{1/2}$ is not a real number.

R.6 Example 6 Using the Rules for Exponents (page 54)

Simplify each expression.

$$(a) \frac{18^{1/2} \cdot 18^{7/2}}{18^3} = \frac{18^{1/2+7/2}}{18^3} = 18^{4-3} = 18^1 = 18$$

$$(b) 100^{3/2} \cdot 16^{-3/4} = (100^{1/2})^3 (16^{1/4})^{-3} = 10^3 \cdot 2^{-3} \\ = 10^3 \cdot \frac{1}{2^3} = \frac{10^3}{2^3} = \frac{1000}{8} = 125$$

$$(c) 4z^{3/4} \cdot 5z^{2/5} = 20z^{3/4+2/5} = 20z^{23/20}$$

R.6 Example 6 Using the Rules for Exponents (cont.)

Simplify each expression.

$$\begin{aligned} \text{(d)} \quad \left(\frac{5m^{4/3}}{n^{2/3}} \right)^2 \left(\frac{m^4}{8n^5} \right)^{1/3} &= \frac{5^2 m^{8/3}}{n^{4/3}} \cdot \frac{m^{4/3}}{8^{1/3} n^{5/3}} \\ &= \frac{25m^{8/3+4/3}}{2n^{4/3+5/3}} = \frac{25m^4}{2n^3} \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad y^{3/7} (y^{4/7} - 5y^{11/7}) &= y^{3/7+4/7} - 5y^{3/7+11/7} \\ &= y - 5y^2 \end{aligned}$$

R.6 Example 7 Factoring Expressions with Negative or Rational Exponents (page 54)

Factor out the least power of the variable or variable expression.

$$(a) 28y^{-5} + 21y^{-2} = 7y^{-5}(4 + 3y^3)$$

$$(b) 18n^{4/3} - 12n^{1/3} = 6n^{1/3}(3n - 2)$$

$$(c) (x + 3)^{-2/5} - (x + 3)^{3/5} = (x + 3)^{-2/5} [1 - (x + 3)] \\ = (x + 3)^{-2/5} (-2 - x)$$

R.6 Example 8 Simplifying a Fraction with Negative Exponents (page 55)

Simplify. Write the result with only positive exponents.

$$\begin{aligned}\frac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}} &= \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x^2} - \frac{1}{y^2}} = \frac{\frac{x+y}{xy}}{\frac{y^2-x^2}{x^2y^2}} \\ &= \frac{x^2y^2 \cdot \frac{x+y}{xy}}{x^2y^2 \cdot \frac{y^2-x^2}{x^2y^2}} = \frac{xy(x+y)}{y^2-x^2} \\ &= \frac{xy(x+y)}{(y-x)(y+x)} = \frac{xy}{y-x}\end{aligned}$$

Divide out the common factor

R.7 Radical Expressions

Radical Notations ■ Simplified Radicals ■ Operations with Radicals ■ Rationalizing Denominators

R.7 Example 1 Evaluating Roots (page 59)

Write each root using exponents and evaluate.

$$(a) \sqrt[3]{27} = 27^{1/3} = 3$$

$$(b) -\sqrt[4]{10,000} = -10,000^{1/4} = -10$$

$$(c) \sqrt[3]{-216} = (-216)^{1/3} = -6$$

$$(d) \sqrt[4]{-81} \text{ is not a real number.}$$

R.7 Example 1 Evaluating Roots (cont.)

Write each root using exponents and evaluate.

$$(e) \sqrt[3]{\frac{125}{512}} = \left(\frac{125}{512}\right)^{1/3} = \frac{125^{1/3}}{512^{1/3}} = \frac{5}{8}$$

$$(f) -\sqrt[5]{-243} = -(-243)^{1/5} = -(-3) = 3$$

R.7 Example 2 Converting From Rational Exponents to Radicals (page 60)

Write in radical form and simplify.

$$(a) 16^{3/4} = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$$

$$(b) (-64)^{2/3} = \left(\sqrt[3]{-64}\right)^2 = (-4)^2 = 16$$

$$(c) -121^{3/2} = -\left(\sqrt{121}\right)^3 = -11^3 = -1331$$

$$(d) y^{7/8} = \sqrt[8]{y^7}, y \geq 0$$

R.7 Example 2 Converting From Rational Exponents to Radicals (cont.)

Write in radical form and simplify.

$$(e) 7z^{4/5} = 7\sqrt[5]{z^4}$$

$$(f) 12q^{-1/4} = \frac{12}{\sqrt[4]{q}}, q > 0$$

$$(g) (5x + 2y)^{1/6} = \sqrt[6]{5x + 2y}$$

R.7 Example 3 Converting From Radicals to Rational Exponents (page 60)

Write in exponential form.

$$(a) \sqrt[7]{n^3} = n^{3/7}$$

$$(b) \sqrt[4]{10x} = (10x)^{1/4}$$

$$(c) 15(\sqrt[3]{r})^4 = 15r^{4/3}$$

$$(d) -2\sqrt[5]{(3x^2)^8} = -2\sqrt[5]{3^8 x^{16}} = -2 \cdot 3^{8/5} x^{16/5}$$

$$(e) \sqrt[3]{r^2 + s^4} = (r^2 + s^4)^{1/3}$$

R.7 Example 4 Using Absolute Value to Simplify Roots (page 61)

Simplify each expression.

$$(a) \sqrt{z^6} = \sqrt{(z^3)^2} = |z^3|$$

$$(b) \sqrt[7]{t^7} = t^{7/7} = t$$

$$(c) \sqrt{81r^8s^{10}} = |9r^4s^5| = 9r^4|s^5|$$

$$(d) \sqrt[4]{(-3)^4} = |-3| = 3$$

R.7 Example 4 Using Absolute Value to Simplify Roots (cont.)

Simplify each expression.

$$(e) \sqrt[5]{m^{10}} = m^{10/5} = m^2$$

$$(f) \sqrt{(3x - 4)^2} = |3x - 4|$$

$$(g) \sqrt{x^2 - 10x + 25} = \sqrt{(x - 5)^2} = |x - 5|$$

R.7 Example 5 Simplifying Radical Expressions (page 62)

Simplify each expression.

$$(a) \sqrt{5} \cdot \sqrt{45} = \sqrt{5 \cdot 45} = \sqrt{225} = 15$$

$$(b) \sqrt[5]{n^3} \cdot \sqrt[5]{n^2} = \sqrt[5]{n^3 \cdot n^2} = \sqrt[5]{n^5} = n$$

$$(c) \sqrt{\frac{11}{169}} = \frac{\sqrt{11}}{\sqrt{169}} = \frac{\sqrt{11}}{13}$$

R.7 Example 5 Simplifying Radical Expressions (cont.)

Simplify each expression.

$$(d) \sqrt[6]{\frac{a}{b^{12}}} = \frac{\sqrt[6]{a}}{\sqrt[6]{b^{12}}} = \frac{\sqrt[6]{a}}{b^2}$$

$$(e) \sqrt[5]{4\sqrt{17}} = \sqrt[20]{17}$$

$$(f) \sqrt{\sqrt[6]{8}} = \sqrt[12]{8}$$

R.7 Example 6 Simplifying Radicals (page 62)

Simplify each radical.

$$(a) \sqrt{288} = \sqrt{2 \cdot 144} = \sqrt{2} \cdot \sqrt{144} = 12\sqrt{2}$$

$$(b) -8\sqrt[3]{125} = -8 \cdot 5 = -40$$

$$\begin{aligned}(c) \sqrt[3]{128a^6b^8c^{10}} &= \sqrt[3]{2 \cdot 64a^6b^6b^2c^9c} \\ &= \sqrt[3]{(64a^6b^6c^9)(2b^2c)} \\ &= 4a^2b^2c^3\sqrt[3]{2b^2c}\end{aligned}$$

R.7 Example 7 Adding and Subtracting Like Radicals (page 63)

Add or subtract as indicated.

$$\begin{aligned} \text{(a)} \quad 14\sqrt{5pq} - 11\sqrt{5pq} &= (14 - 11)\sqrt{5pq} \\ &= 3\sqrt{5pq} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \sqrt{75ab^3} - b\sqrt{12ab} &= \sqrt{3 \cdot 25ab^2b} - b\sqrt{4 \cdot 3ab} \\ &= 5b\sqrt{3ab} - 2b\sqrt{3ab} \\ &= (5b - 2b)\sqrt{3ab} \\ &= 3b\sqrt{3ab} \end{aligned}$$

R.7 Example 7 Adding and Subtracting Like Radicals (cont.)

Add or subtract as indicated.

$$\begin{aligned} \text{(c)} \quad & \sqrt[3]{81x^5y^7} + \sqrt[3]{24x^8y^4} \\ &= \sqrt[3]{27 \cdot 3x^3x^2y^6y} + \sqrt[3]{8 \cdot 3x^6x^2y^3y} \\ &= 3xy^2\sqrt[3]{3x^2y} + 2x^2y\sqrt[3]{3x^2y} \\ &= (3xy^2 + 2x^2y)\sqrt[3]{3x^2y} \end{aligned}$$

R.7 Example 8 Simplifying Radicals (page 64)

Simplify each radical.

$$(a) \sqrt[10]{2^5} = 2^{5/10} = 2^{1/2} = \sqrt{2}$$

$$(b) \sqrt[3]{a^9 b^{18}} = a^{9/3} b^{18/3} = a^3 b^6$$

$$(c) \sqrt[6]{\sqrt[3]{4^2}} = \sqrt[18]{4^2} = 4^{2/18} = 4^{1/9} = \sqrt[9]{4}$$

R.7 Example 9(a) Multiplying Radical Expressions (page 64)

Find the product.

$$\begin{aligned}(\sqrt{11} + \sqrt{17})(\sqrt{11} - \sqrt{17}) &= (\sqrt{11})^2 - (\sqrt{17})^2 \\ &= 11 - 17 = -6\end{aligned}$$

Product of the sum and difference of two terms.

R.7 Example 9(b) Multiplying Radical Expressions (page 64)

Find the product.

$$\begin{aligned}(5 + \sqrt{32})(3 - \sqrt{2}) &= (5 + \sqrt{2 \cdot 16})(3 - \sqrt{2}) && \text{Simplify } \sqrt{32} \\ &= (5 + 4\sqrt{2})(3 - \sqrt{2}) \\ &= 15 - 5\sqrt{2} + 12\sqrt{2} - 4\sqrt{2}\sqrt{2} \\ & && \text{FOIL} \\ &= 15 - 5\sqrt{2} + 12\sqrt{2} - 8 \\ &= 7 + 7\sqrt{2}\end{aligned}$$

R.7 Example 10 Rationalizing Denominators (page 65)

Rationalize each denominator.

$$(a) \frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$$

$$(b) \sqrt[3]{\frac{4}{9}} = \frac{\sqrt[3]{4}}{\sqrt[3]{9}} = \frac{\sqrt[3]{4} \cdot \sqrt[3]{3}}{\sqrt[3]{3^2} \cdot \sqrt[3]{3}} = \frac{\sqrt[3]{12}}{\sqrt[3]{3^3}} = \frac{\sqrt[3]{12}}{3}$$

R.7 Example 11(a) Simplifying Radical Expressions with Fractions (page 65)

Simplify the expression.

$$\frac{\sqrt[3]{a^5 b}}{\sqrt[3]{a^2 b^5}} = \sqrt[3]{\frac{a^5 b}{a^2 b^5}} = \sqrt[3]{\frac{a^3}{b^4}} = \frac{\sqrt[3]{a^3}}{\sqrt[3]{b^4}}$$

Quotient rule

$$= \frac{a}{b\sqrt[3]{b}} = \frac{a\sqrt[3]{b^2}}{b\sqrt[3]{b}\sqrt[3]{b^2}}$$

Rationalize denominator.

$$= \frac{a\sqrt[3]{b^2}}{b\sqrt[3]{b^3}} = \frac{a\sqrt[3]{b^2}}{b \cdot b} = \frac{a\sqrt[3]{b^2}}{b^2}$$

R.7 Example 11(b) Simplifying Radical Expressions with Fractions (page 65)

Simplify the expression.

$$\begin{aligned}\sqrt[4]{\frac{6}{x^8}} - \sqrt[4]{\frac{3}{x^{16}}} &= \frac{\sqrt[4]{6}}{\sqrt[4]{x^8}} - \frac{\sqrt[4]{3}}{\sqrt[4]{x^{16}}} \\ &= \frac{\sqrt[4]{6}}{x^2} - \frac{\sqrt[4]{3}}{x^4} \\ &= \frac{x^2 \sqrt[4]{6}}{x^4} - \frac{\sqrt[4]{3}}{x^4} \\ &= \frac{x^2 \sqrt[4]{6} - \sqrt[4]{3}}{x^4}\end{aligned}$$

Quotient rule

Simplify the denominators.

Write with a common denominator.

Subtract the numerators.

R.7 Example 12 Rationalizing a Binomial Denominator

(page 66)

Rationalize the denominator.

$$\begin{aligned}\frac{2}{3 + \sqrt{5}} &= \frac{2(3 - \sqrt{5})}{(3 + \sqrt{5})(3 - \sqrt{5})} \\ &= \frac{2(3 - \sqrt{5})}{3^2 - (\sqrt{5})^2} = \frac{2(3 - \sqrt{5})}{9 - 5} \\ &= \frac{2(3 - \sqrt{5})}{4} = \frac{3 - \sqrt{5}}{2}\end{aligned}$$

Multiply the numerator and denominator by the conjugate of the denominator.