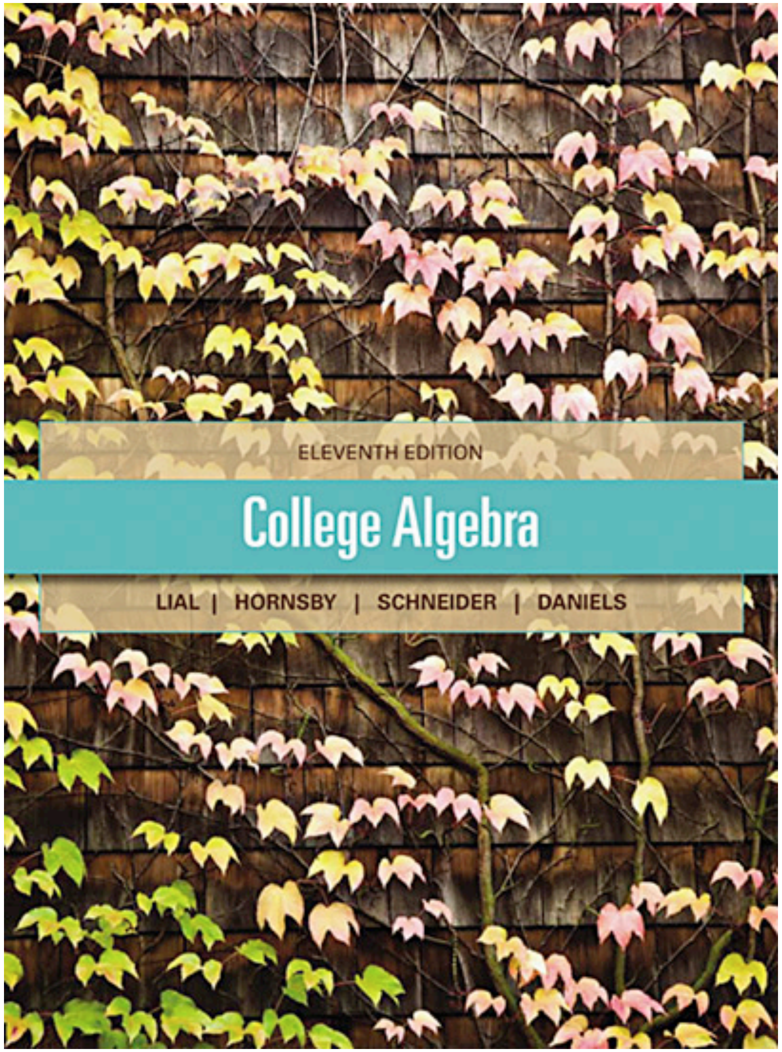
R

Review of Basic Concepts

Sections R.1–R.4



R Review of Basic Concepts

R.1 Sets

Basic Definitions ▪ Operations on Sets

Example 1 Using Set Notation and Terminology (page 2)

Identify each set as *finite* or *infinite*. Then determine whether 8 is an element of the set.

(a) {5, 6, 7, …, 10}

finite; 8 is an element.

(b) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$

{infinite; 8 is not an element. }

Example 1 Using Set Notation and Terminology (cont)

Identify each set as *finite* or *infinite*. Then determine whether 8 is an element of the set.

(c) $\{x \mid x \text{ is a fraction between 9 and 10}\}$

infinite; 8 is not an element.

(d) $\{x \mid x \text{ is a natural number between 7 and 9}\}$

{finite; 8 is an element. }

R.1 Example 2 Listing the Elements of a Set (page 3)

Write the elements belonging to:

(a) $\{x|x$ is a natural number between 8 and 12$\}$

$\{9, 10, 11\}$

(b) $\{x|x$ is a natural number greater than 6 and less than 8$\}$

$\{7\}$

Example 3 Examining Subset Relationships (page 4)

Let $U = \{3, 9, 15, 21, 27, 33, 39\}$,
$A = \{3, 9, 15, 21, 27, 33\}$, $B = \{3, 9, 21, 27\}$,
$C = \{9, 27, 33\}$, and $D = \{3, 27\}$. Determine whether each statement is *true* or *false.*

(a) $B \subseteq D$

false

(b) $D \subseteq B$

true

Example 3 Examining Subset Relationships (cont)

Let $U = \{3, 9, 15, 21, 27, 33, 39\}$,
$A = \{3, 9, 15, 21, 27, 33\}$, $B = \{3, 9, 21, 27\}$,
$C = \{9, 27, 33\}$, and $D = \{3, 27\}$. Determine whether each statement is *true* or *false.*

(c) $D \not\subseteq A$

false

(d) $U \neq A$

true

Example 4 Finding the Complement of a Set (page 4)

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$,

$A = \{2, 4, 6, 8\}$, $B = \{3, 6, 9\}$

Find A', B', U', $\varnothing'$

A' contains the elements of U that are not in A:
$\{1, 3, 5, 7, 9\}$

B' contains the elements of U that are not in B:
$\{1, 2, 4, 5, 7, 8\}$

$U' = \varnothing$ $\qquad$ $\varnothing' = U$

Example 5 Finding the Intersection of Two Sets (page 5)

Find

(a) $\{15, 20, 25, 30\} \cap \{12, 18, 24, 30\}$

$\{15, 20, 25, 30\} \cap \{12, 18, 24, 30\} = \{30\}$

The element 30 is the only one belonging to both sets.

Example 5 Finding the Intersection of Two Sets (cont.)

Find

(b) $\{3, 6, 9, 12, 15, 18\} \cap \{6, 12, 18, 24\}$

$\{3, 6, 9, 12, 15, 18\} \cap \{6, 12, 18, 24\} = \{6, 12, 18\}$

The elements 6, 12, and 18 belong to both sets.

Example 5 Finding the Intersection of Two Sets (cont.)

Find

(c) $\{6, 7, 8\} \cap \{678\}$

$\{6, 7, 8\} \cap \{678\}$ = no solution; disjoint sets

R.1 Example 6 Finding the Union of Two Sets (page 5)

Find

(a) $\{1, 3, 5, 7, 9\} \cup \{3, 6, 9, 12\}$

List the elements of the first set, then include the elements from the second set that are not already listed.

$\{1, 3, 5, 7, 9\} \cup \{3, 6, 9, 12\} = \{1, 3, 5, 6, 7, 9, 12\}$

Example 6 Finding the Union of Two Sets (cont.)

Find

(b) {9, 10, 11, 12} U {10, 12, 14, 16}

{9, 10, 11, 12} U {10, 12, 14, 16}
= {9, 10, 11, 12, 14, 16}

R.1 Example 6 Finding the Union of Two Sets (cont.)

Find

(c) {2, 6, 10, 14, …} U {4, 8, 12, 16, …}

{2, 4, 6, 8, 10, …}

R.2 Real Numbers and Their Properties

Sets of Numbers and the Number Line ▪ Exponents ▪ Order of Operations ▪ Properties of Real Numbers ▪ Order on the Number Line ▪ Absolute Value

R.2 Example 1 Identifying Sets of Numbers (page 9)

$$\text{Let } S = \left\{-12, -7.25, -\sqrt{3}, -\frac{5}{6}, 0, .\overline{45}, \sqrt{9}, \frac{21}{3}, 999\right\}$$

List the elements of *S* that belong to each set.

(a) natural numbers $\left\{\sqrt{9} \text{ (or 3)}, \frac{21}{3} \text{ (or 7)}, 999\right\}$

(b) whole numbers $\left\{0, \sqrt{9} \text{ (or 3)}, \frac{21}{3} \text{ (or 7)}, 999\right\}$

Example 1 Identifying Sets of Numbers (cont.)

$$\text{Let } S = \left\{-12, -7.25, -\sqrt{3}, -\frac{5}{6}, 0, .\overline{45}, \sqrt{9}, \frac{21}{3}, 999\right\}$$

List the elements of *S* that belong to each set.

(c) integers $\left\{-12, 0, \sqrt{9}\text{ (or 3)}, \frac{21}{3}\text{ (or 7)}, 999\right\}$

(d) rational numbers All elements of *S* except $-\sqrt{3}$

Example 1 Identifying Sets of Numbers (cont.)

Let $S = \left\{-12, -7.25, -\sqrt{3}, -\frac{5}{6}, 0, .\overline{45}, \sqrt{9}, \frac{21}{3}, 999\right\}$

List the elements of *S* that belong to each set.

(e) irrational numbers $-\sqrt{3}$

(f) real numbers All elements of *S* are real numbers.

Example 2 Evaluating Exponential Expressions (page 10)

Evaluate each expression and identify the base and the exponent.

(a) 10^3 $10^3 = \underbrace{10 \cdot 10 \cdot 10}_{3 \text{ factors of } 10} = 1000$

Base: 10

Exponent: 3

(b) $(-3)^4$ $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$

Base: –3 Exponent: 4

Example 2 Evaluating Exponential Expressions (cont.)

Evaluate each expression and identify the base and the exponent.

(c) -3^4

$$-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$$

Base: 3 Exponent: 4

(d) $2 \cdot 5^2$

$$2 \cdot 5^2 = 2 \cdot 5 \cdot 5 = 50$$

Base: 5 Exponent: 2

(e) $(2 \cdot 5)^2$

$$(2 \cdot 5)^2 = 10^2 = 10 \cdot 10 = 100$$

Base: 10 Exponent: 2

R.2 Example 3(a) Using Order of Operations (page 10)

Evaluate $3 \cdot 9 - 2^5 \div 4$

$$3 \cdot 9 - 2^5 \div 4 = 3 \cdot 9 - 32 \div 4$$ Evaluate the exponential

$$= 27 - 32 \div 4$$ Multiply

$$= 27 - 8$$ Divide

$$= 19$$ Subtract

Example 3(b) Using Order of Operations (cont.)

Evaluate $(30-5)\cdot 3 \div 15 + 7$

$$(30-5)\cdot 3 \div 15 + 7 = 25 \cdot 3 \div 15 + 7$$

Work within the parenthesis

$$= 75 \div 15 + 7$$

Multiply

$$= 5 + 7$$

Divide

$$= 12$$

Add

Example 3(c) Using Order of Operations (cont.)

Evaluate $\dfrac{2^4 - 11}{9 + 3 \cdot 2}$

$$\frac{2^4 - 11}{9 + 3 \cdot 2} = \frac{16 - 11}{9 + 6}$$

Evaluate the exponential

Multiply

$$= \frac{5}{15}$$

Subtract

Add

$$= \frac{1}{3}$$

Simplify

Example 3(d) Using Order of Operations (cont.)

Evaluate $\dfrac{-7^2-(-9)}{6(-3)-1(-2)}$

$$\frac{-7^2-(-9)}{6(-3)-1(-2)}=\frac{-49-(-9)}{6(-3)-1(-2)}$$

Evaluate the exponential

$$=\frac{-49+9}{-18+2}$$

Multiply

$$=\frac{-40}{-16}=\frac{5}{2}$$

Add in numerator and denominator, then simplify.

Example 4(a) Using Order of Operations (page 11)

Evaluate $6x^2 + 5y - 3z$ using $x = -4$, $y = 3$, and $z = -6$.

$$6x^2 + 5y - 3z = 6(-4)^2 + 5(3) - 3(-6)$$ Substitute

$$= 6(16) + 5(3) - 3(-6)$$ Evaluate the exponential

$$= 96 + 15 + 18$$ Multiply

$$= 129$$ Add

R.2 Example 4(b) Using Order of Operations (cont.)

Evaluate $\dfrac{4y-3(x-1)^2}{z+9}$ using $x = -4$, $y = 3$, and $z = -6$.

$$\frac{4y-3(x-1)^2}{z+9} = \frac{4(3)-3(-4-1)^2}{-6+9}$$

Substitute

$$= \frac{4(3)-3(-5)^2}{-6+9}$$

Subtract

$$= \frac{4(3)-3(25)}{-6+9}$$

Evaluate the exponential

$$= \frac{12-75}{3} = \frac{-63}{3} = -21$$

Multiply, subtract, and simplify

Example 4(c) Using Order of Operations (cont.)

Evaluate $\dfrac{\dfrac{x}{4}+\dfrac{y}{3}}{\dfrac{z}{2}-\dfrac{x}{2}}$ using $x = -4$, $y = 3$, and $z = -6$.

$$\frac{\dfrac{x}{4}+\dfrac{y}{3}}{\dfrac{z}{2}-\dfrac{x}{2}} = \frac{\dfrac{-4}{4}+\dfrac{3}{3}}{\dfrac{-6}{2}-\dfrac{-4}{2}}$$

$$= \frac{-1+1}{-3-(-2)}$$

$$= \frac{0}{-1} = 0$$

R.2 **Example 5(a)** Simplifying Expressions (page 13)

Simplify $(12 + 2x) + 18$.

$$(12 + 2x) + 18 = (2x + 12) + 18$$ Commutative property

$$= 2x + (12 + 18)$$ Associative property

$$= 2x + 30$$

R.2 Example 5(b) Using the Commutative and Associative Properties to Simplify Expressions (cont.)

Simplify $\left(\frac{4}{7}\right)(-35t)$

$$\left(\frac{4}{7}\right)(-35t) = \left(\frac{4}{7}\cdot(-35)\right)t$$

Associative property

$$= -20t$$

R.2 Example 5(c) Using the Commutative and Associative Properties to Simplify Expressions (cont.)

Simplify $(-54s)\left(-\frac{4}{9}\right)$

$$(-54s)\left(-\frac{4}{9}\right) = \left(s(-54)\right)\left(-\frac{4}{9}\right)$$ Commutative property

$$= s\left(-54 \cdot \left(-\frac{4}{9}\right)\right)$$ Associative property

$$= \left(-54 \cdot \left(-\frac{4}{9}\right)\right)s$$ Commutative property

$$= 24s$$

Example 6 Using the Distributive Property (page 13)

Rewrite using the distributive property and simplify.

(a) $8(m - 2n) = 8(m - 2n) = 8m - 16n$

(b) $-(-3r + 5s) = -1(-3r + 5s) = 3r - 5s$

Rewrite using the distributive property and simplify.

(c) $\frac{3}{4}\left(\frac{5}{6}p + \frac{1}{2}q - 28\right)$

$$= \frac{3}{4}\left(\frac{5}{6}p + \frac{1}{2}q - 28\right)$$

$$= \frac{3}{4}\left(\frac{5}{6}p\right) + \frac{3}{4}\left(\frac{1}{2}q\right) - \frac{3}{4}(28)$$

$$= \frac{5}{8}p + \frac{3}{8}q - 21$$

Example 6 Using the Distributive Property (cont.)

Rewrite using the distributive property and simplify.

(d) $22t - 55 = 11(2t) - 11(5) = 11(2t - 5)$

R.2 Example 7 Evaluating Absolute Values (page 14)

Evaluate each expression:

(a) $|-6.85| = 6.85$

(b) $-|50| = -50$

(c) $-\left|-\dfrac{2}{3}\right| = -\dfrac{2}{3}$

(d) $|y|$, if $y = \sqrt{2}$

$|\sqrt{2}| = \sqrt{2}$

R.2 Example 8 Measuring Blood Pressure Difference (page 15)

Find P_d for a patient with a systolic pressure, P, of 146.

$$P_d = |P - 120| = |146 - 120| = |26| = 26$$

Let *m* = 13 and *n* = –9. Evaluate each expression.

(a) $$|3m+5n| = |3(13)+5(-9)| = |39-45| = |-6| = 6$$

(b) $$\frac{|2m|-3|n|}{|m+n|} = \frac{|2(13)|-3|(-9)|}{|13+(-9)|} = \frac{|26|-3|-9|}{|4|} = \frac{26-3(9)}{4} = \frac{26-27}{4} = -\frac{1}{4}$$

R.2 Example 10 Finding the Distance between Two Points (page 16)

Find the distance between –8 and 14.

$$|b - a| = |14 - (-8)|$$

$$= |14 + 8|$$

$$= |22| = 22$$

R.3 Polynomials

Rules for Exponents ▪ Polynomials ▪ Addition and Subtraction ▪ Multiplication ▪ Division

Example 1 Using the Product Rule (page 21)

Find each product.

(a) $m^6 \cdot m^8 = m^{6+8} = m^{14}$

(b) $\left(-5r^3\right)\left(6r^4\right)(-3r) = (-5)(6)(-3)\cdot\left(r^3 r^4 r\right)$

Commutative and associative properties

$= 90 \cdot r^{3+4+1}$ Product rule

$= 90r^8$

Example 2 Using the Power Rules (page 22)

Simplify. Assume all variables represent nonzero real numbers.

(a) $\left(7^{3}\right)^{5}=7^{3(5)}=7^{15}$

(b) $\left(2^{5}y^{3}\right)^{4}=\left(2^{5}\right)^{4}\left(y^{3}\right)^{4}$

$=2^{5(4)}y^{3(4)}=2^{20}y^{12}$

Simplify. Assume all variables represent nonzero real numbers.

(c) $$\left(\frac{4^3}{z^2}\right)^5 = \frac{\left(4^3\right)^5}{\left(z^2\right)^5} = \frac{4^{3(5)}}{z^{2(5)}} = \frac{4^{15}}{z^{10}}$$

(d) $$\left(\frac{-3a^3}{bc^4}\right)^2 = \frac{\left(-3a^3\right)^2}{\left(bc^4\right)^2} = \frac{(-3)^2\left(a^3\right)^2}{b^2\left(c^4\right)^2} = \frac{9a^6}{b^2c^8}$$

R.3 Example 3 Using the Definition of a^0 (page 23)

Evaluate each power.

(a) 8^0 (b) -8^0 (c) $(-8)^0$

(d) $-(-8)^0$ (e) $(-3b^8)^0$

(a) $8^0 = 1$ (b) $-8^0 = -1$ (c) $(-8)^0 = 1$

(d) $-(-8)^0 = -1$ (e) $(-3b^8)^0 = 1$, $b \neq 0$

Example 4 Classifying Polynomials (page 24)

Fill in the blank entries with the appropriate response.

Polynomial	Degree	Type
$x^4 + 3x^3 + 5$	4	trinomial
$16x^2y^5$	7	monomial
$m^3 - n^3$	3	binomial

Example 5 Adding and Subtracting Polynomials (page 24)

Add or subtract.

(a) $\left(17x^3 - 10x^2 + x\right) + \left(-9x^3 + 10x^2 - 5x\right)$

$$= (17 - 9)x^3 + (-10 + 10)x^2 + (1 - 5)x$$

$$= 8x^3 - 4x$$

(b) $\left(-6m^4 - 11m^2 + 21\right) - \left(m^4 - 6m^2 + 35\right)$

$$= (-6 - 1)m^4 + \left[-11 - (-6)\right]m^2 + (21 - 35)$$

$$= -7m^4 - 5m^2 - 14$$

R.3 Example 5 Adding and Subtracting Polynomials (cont.)

Add or subtract.

(c) $\left(10r^3s^6 + 5r^6s^3\right) + \left(25r^3s^6 - 15r^6s^3\right)$

$$= (10 + 25)r^3s^6 + (5 - 15)r^6s^3$$

$$= 35r^3s^6 - 10r^6s^3$$

Add or subtract.

(d) $6\left(z^2 - 5z + 3\right) - 4\left(3z^2 - 2z + 9\right)$

$= 6z^2 - 6(5z) + 6(3) - 4\left(3z^2\right) - 4(-2z) - 4(9)$

$= 6z^2 - 30z + 18 - 12z^2 + 8z - 36$

$= -6z^2 - 22z - 18$

R.3 Example 6 Multiplying Polynomials (page 25)

Multiply $(4t-5)(3t^2-2t+7)$

$$\begin{array}{rrrrl}
 & 3t^2 & -2t & +7 & \\
 & & 4t & -5 & \\
\hline
 & -15t^2 & +10t & -35 & \leftarrow -5(3t^2-2t+7) \\
12t^3 & -8t^2 & +28t & & \leftarrow 4t(3t^2-2t+7) \\
\hline
12t^3 & -23t^2 & +38t & -35 & \text{Add in columns}
\end{array}$$

Example 7(a) Using FOIL to Multiply Two Binomials (page 25)

Find the product.

$$(7y+3)(4y-5)$$

$$\begin{array}{l} \qquad\quad \text{F} \qquad\qquad\quad \text{O} \qquad\qquad\quad \text{I} \qquad\qquad \text{L} \\ =(7y)(4y)+(7y)(-5)+3(4y)+3(-5) \\ =28y^2-23y-15 \end{array}$$

$-35y+12y=-23y$

Example 7(b) Using FOIL to Multiply Two Binomials (cont.)

Find the product.

$$(6p+11)(6p-11)$$

$$\begin{array}{l} \qquad\quad \text{F} \qquad\qquad \text{O} \qquad\qquad\qquad \text{I} \qquad\qquad\qquad \text{L} \\ =(6p)(6p)+6p(-11)+11(6p)+11(-11) \end{array}$$

$$=36p^2-121 \qquad -66p+66p=0$$

R.3 Example 7(c) Using FOIL to Multiply Two Binomials (cont.)

Find the product.

$$x^3(2x-5)(2x+5)$$

$$= x^3\left[2x(2x)+2x(5)+(-5)(2x)+(-5)(5)\right] \quad \text{FOIL}$$

$$= x^3\left(4x^2+10x-10x-25\right) = x^3\left(4x^2-25\right)$$

Combine like terms

$$= 4x^5-25x^3 \quad \text{Distributive property}$$

Example 8 Using the Special Products (page 26)

Find each product.

(a) $(7m-10)(7m+10) = 49m^2 - 100$

(b) $(4r^2+9)(4r^2-9) = 16r^4 - 81$

(c) $(5x^2-8y^4)(5x^2+8y^4) = 25x^4 - 64y^8$

Example 8 Using the Special Products (cont.)

Find each product.

(d) $(8z+3)^2 = 64z^2 + 48z + 9$

(e) $\left(5z - 12q^3\right)^2 = 25z^2 - 120zq^3 + 144q^6$

Example 9(a) Multiplying More Complicated Binomials (page 27)

Find the product: $\left[(4x-3)+7y\right]\left[(4x-3)-7y\right]$

$$\left[(4x-3)+7y\right]\left[(4x-3)-7y\right]$$

Product of the sum and difference of two terms

$$=(4x-3)^2-(7y)^2$$

$$=16x^2-24x+9-49y^2$$

Example 9(b) Multiplying More Complicated Binomials (cont.)

Find the product: $(a-b)^4$

$$(a-b)^4 = (a-b)^2(a-b)^2$$

$$= \left(a^2 - 2ab + b^2\right)\left(a^2 - 2ab + b^2\right)$$

$$= a^4 - 2a^3b + a^2b^2 - 2a^3b + 4a^2b^2 - 2ab^3 + a^2b^2 - 2ab^3 + b^4$$

$$= a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

Example 9(c) Multiplying More Complicated Binomials (cont.)

Find the product: $(s+4t)^3$

$$\begin{aligned}(s+4t)^3 &= (s+4t)^2(s+4t)\\ &= \left(s^2+8st+16t^2\right)(s+4t)\\ &= s^3+8s^2t+16st^2+4s^2t+32st^2+64t^3\\ &= s^3+12s^2t+48st^2+64t^3\end{aligned}$$

Example 10 Dividing Polynomials (page 27)

Divide $12n^3 + 11n^2 + 5n - 8$ by $3n + 2$

$$
\begin{array}{r}
4n^2 + n + 1 \\
3n+2 \overline{\smash{\big)}\, 12n^3 + 11n^2 + 5n - 8} \\
\underline{12n^3 + 8n^2} \\
3n^2 + 5n \\
\underline{3n^2 + 2n} \\
3n - 8 \\
\underline{3n + 2} \\
-10
\end{array}
$$

Example 10 Dividing Polynomials (cont.)

$$\left(12n^3 + 11n^2 + 5n - 8\right) \div \left(3n + 2\right) = 4n^2 + n + 1 + \frac{-10}{3n + 2}$$

Example 11 Dividing Polynomials with Missing Terms (page 28)

Divide $8x^4 + 12x^2 + 7x - 18$ by $x^2 + 2$

$$
\begin{array}{r}
8x^2 \qquad\qquad - 4 \\
x^2 + 0x + 2 \,\overline{\big)\, 8x^4 + 0x^3 + 12x^2 + 7x - 18} \\
\underline{8x^4 + 0x^3 + 16x^2} \qquad\qquad\quad \\
-4x^2 + 7x - 18 \\
\underline{-4x^2 - 0x - \ \ 8} \\
7x - 10
\end{array}
$$

$$\left(8x^4 + 12x^2 + 7x - 18\right) \div \left(x^2 + 2\right) = 8x^2 - 4 + \frac{7x - 10}{x^2 + 2}$$

R.4 Factoring Polynomials

Factoring Out the Greatest Common Factor ▪ Factoring by Grouping ▪ Factoring Trinomials ▪ Factoring Binomials ▪ Factoring by Substitution

Example 1 Factoring Out the Greatest Common Factor (page 32)

Factor out the greatest common factor from each polynomial.

(a) $6a^2 - 18a^4 = 6a^2\left(1 - 3a^2\right)$

(b) $14x^3y^2 - 28x^2y^3 + 21x^2y^2$

$= 7x^2y^2\left(2x - 4y + 3\right)$

Example 1 Factoring Out the Greatest Common Factor (cont.)

Factor out the greatest common factor from the polynomial.

(c) $24(x-2)^3 - 16(x-2)^2 + 6(x-2)$

$$= 2(x-2)\left[12(x-2)^2 - 8(x-2) + 1\right]$$

GCF = 2(*x* – 2)

$$= 2(x-2)\left[12\left(x^2 - 4x + 4\right) - 8x + 16 + 3\right]$$

$$= 2(x-2)\left(12x^2 - 48x + 48 - 8x + 16 + 3\right)$$

$$= 2(x-2)\left(12x^2 - 56x + 67\right)$$

Example 2(a) Factoring By Grouping (page 33)

Factor by grouping.

$$r^2s + 3r^2 - 5s - 15 = (r^2s + 3r^2) - (5s + 15)$$

$$= r^2(s + 3) - 5(s + 3)$$

$$= (r^2 - 5)(s + 3)$$

Example 2(b) Factoring By Grouping (page 33)

Factor by grouping.

$$4m^2 - m^2n + 4n - n^2 = (4m^2 - m^2n) + (4n - n^2)$$

$$= m^2(4 - n) + n(4 - n)$$

$$= (m^2 + n)(4 - n)$$

Example 2(c) Factoring By Grouping (page 33)

Factor by grouping.

$$9y^3 - 15y^2 + 6y - 10 = (9y^3 - 15y^2) + (6y - 10)$$

$$= 3y^2(3y - 5) + 2(3y - 5)$$

$$= (3y^2 + 2)(3y - 5)$$

Example 3(a) Factoring Trinomials (page 34)

Factor $5z^2 + 4z - 12$, if possible.

The positive factors of 5 are 5 and 1.

The factors of –12 are –12 and 1, 12 and – 1, –6 and 2, 6 and –2, –4 and 3, or 4 and –3.

Example 3(a) Factoring Trinomials (cont.)

Factor $5z^2 + 4z - 12$.

Try different combinations:

$(5z - 12)(z + 1) = 5z^2 - 7z - 12$ incorrect

$(5z + 4)(z - 3) = 5z^2 - 11z - 12$ incorrect

$(5z - 6)(z + 2) = 5z^2 + 4z - 12$ correct

$$5z^2 + 4z - 12 = (5z - 6)(z + 2)$$

Example 3(b) Factoring Trinomials (page 34)

Factor $12t^2 - 5t - 3$, if possible.

The positive factors of 12 are 12 and 1, 6 and 2, or 4 and 3.

The factors of –3 are –3 and 1 or 3 and –1.

Example 3(b) Factoring Trinomials (cont.)

Factor $12t^2 - 5t - 3$.

Try different combinations:

$$(2t + 3)(6t - 1) = 12t^2 + 16t - 3$$ incorrect

$$(12t - 3)(t + 1) = 12t^2 + 9t - 3$$ incorrect

$$(4t - 3)(3t + 1) = 12t^2 - 5t - 3$$ correct

$$12t^2 - 5t - 3 = (4t - 3)(3t + 1)$$

Example 3(c) Factoring Trinomials (page 34)

Factor $3x^2 - 15x + 16$, if possible.

The positive factors of 3 are 3 and 1.

The negative factors of 16 are –16 and – 1,
– 8 and – 2, or – 4 and –4.

Example 3(c) Factoring Trinomials (cont)

Factor $3x^2 - 15x + 16$.

Try different combinations:

$(3x - 16)(x - 1) = 3x^2 - 19x + 16$ incorrect

$(3x - 1)(x - 16) = 3x^2 - 49x + 16$ incorrect

$(3x - 8)(x - 2) = 3x^2 - 14x + 16$ incorrect

$(3x - 2)(x - 8) = 3x^2 - 26x + 16$ incorrect

$(3x - 4)(x - 4) = 3x^2 - 16x + 16$ incorrect

$3x^2 - 15x + 16$ is prime.

Factor $24x^2 + 42x + 15$, if possible.

Factor out the GCF, 3, first:

$$24x^2 + 42x + 15 = 3\left(8x^2 + 14x + 5\right)$$

The positive factors of 8 are 8 and 1 or 4 and 2.

The positive factors of 5 are 5 and 1.

Example 3(d) Factoring Trinomials (cont)

Factor $24x^2 + 42x + 15$.

Try different combinations:

$(2x+5)(4x+1) = 8x^2 + 22x + 5$ incorrect

$(8x+5)(x+1) = 8x^2 + 13x + 5$ incorrect

$(4x+5)(2x+1) = 8x^2 + 14x + 5$ correct

$$24x^2 + 42x + 15 = 3\left(8x^2 + 14x + 5\right) = 3(4x+5)(2x+1)$$

R.4 Example 4 Factoring Perfect Square Trinomials (page 35)

Factor each trinomial:

(a) $49x^2 + 28xy + 4y^2 = (7x + 2y)^2$

(b) $81a^2b^2 - 90ab + 25 = (9ab - 5)^2$

Example 5 Factoring Differences of Squares (page 36)

Factor each trinomial:

(a) $64r^2 - 49 = (8r - 7)(8r + 7)$

(b) $169u^6 - 144v^4 = \left(13u^3 + 12v^2\right)\left(13u^3 - 12v^2\right)$

(c) $(2c - 3d)^2 - 16f^2 = (2c - 3d + 4f)(2c - 3d - 4f)$

Factor the trinomial:

(d) $x^2 + 18x + 81 - 25y^2$

$$= \left(x^2 + 18x + 81\right) - 25y^2$$

$$= (x+9)^2 - 25y^2$$

$$= (x+9+5y)(x+9-5y)$$

Factor the trinomial:

(e) $4x^2 - y^2 - 10y - 25 = 4x^2 - \left(y^2 + 10y + 25\right)$

$$= 4x^2 - (y+5)^2$$

$$= \left(2x + (y+5)\right)\left(2x - (y+5)\right)$$

$$= (2x + y + 5)(2x - y - 5)$$

R.4 Example 6 Factoring Sums or Differences of Cubes (page 37)

Factor each polynomial:

(a) $t^3 + 1000 = t^3 + 10^3 = (t + 10)(t^2 - 10t + 10^2)$

$$= (t + 10)(t^2 - 10t + 100)$$

(b) $r^3 - 8s^3 = r^3 - (2s)^3$

$$= (r - 2s)\left[r^2 + r(2s) + (2s)^2\right]$$

$$= (r - 2s)(r^2 + 2rs + 4s^2)$$

Factor each polynomial:

(c) $125u^9 - 216v^{12}$

$$= \left(5u^3\right)^3 - \left(6v^4\right)^3$$

$$= \left(5u^3 - 6v^4\right)\left[\left(5u^3\right)^2 + \left(5u^3\right)\left(6v^4\right) + \left(6v^4\right)^2\right]$$

$$= \left(5u^3 - 6v^4\right)\left(25u^6 + 30u^3v^4 + 36v^8\right)$$

Example 7(a) Factoring by Substitution (page 37)

Factor $8(3x+1)^2+10(3x+1)-25$

Replace (3*x* + 1) with *u*:

$$8(3x+1)^2+10(3x+1)-25=8u^2+10u-25$$

Factor using FOIL: $8u^2+10u-25=(2u+5)(4u-5)$

Replace *u* with (3*x* + 1), then simplify:

$$\begin{aligned}(2u+5)(4u-5)&=\left[2(3x+1)+5\right]\left[4(3x+1)-5\right]\\&=(6x+2+5)(12x+4-5)\\&=(6x+7)(12x-1)\end{aligned}$$

$$8(3x+1)^2+10(3x+1)-25=(6x+7)(12x-1)$$

Example 7(b) Factoring by Substitution (page 37)

Factor $(3x+1)^3 - 27$

Replace (3*x* + 1) with *u*: $(3x+1)^3 - 27 = u^3 - 27$

Write as the difference of cubes, then factor:

$$u^3 - 27 = u^3 - 3^3 = (u-3)\left(u^2 + 3u + 9\right)$$

Replace *u* with (3*x* + 1), then simplify:

$$\begin{aligned}(u-3)\left(u^2+3u+9\right) &= \left[(3x+1)-3\right]\left[(3x+1)^2 + 3(3x+1) + 9\right] \\ &= (3x-2)\left[9x^2 + 6x + 1 + 9x + 3 + 9\right] \\ &= (3x-2)\left(9x^2 + 15x + 13\right)\end{aligned}$$

$$(3x+1)^3 - 27 = (3x-2)\left(9x^2 + 15x + 13\right)$$

Example 7(c) Factoring by Substitution (page 37)

Factor $15m^4 - m^2 - 6$

Replace m^2 with u: $15m^4 - m^2 - 6 = 15u^2 - u - 6$

Factor using FOIL: $15u^2 - u - 6 = (3u - 2)(5u + 3)$

Replace u with m^2:

$$(3u - 2)(5u + 3) = (3m^2 - 2)(5m^2 + 3)$$

$$15m^4 - m^2 - 6 = (3m^2 - 2)(5m^2 + 3)$$