EXPONENTIAL & Logarithmic FUNCTIONS

5.6

Law	Example
$x^1 = x$	$6^1 = 6$
$x^0 = 1$	$7^0 = 1$
$x^{-1} = 1/x$	$4^{-1} = 1/4$
$x^m x^n = x^{m+n}$	$x^2 x^3 = x^{2+3} = x^5$
$x^m/x^n = x^{m-n}$	$x^6/x^2 = x^{6-2} = x^4$
$(x^m)^n = x^{mn}$	$(x^2)^3 = x^{2\times3} = x^6$
$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
$(x/y)^n = x^n/y^n$	$(x/y)^2 = x^2 / y^2$
$x^{-n} = 1/x^n$	$x^{-3} = 1/x^3$
And the law about Fractional Exponents:	
$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$	$x^{\frac{2}{3}} = \sqrt[3]{x^2} = (\sqrt[3]{x})^2$

Properties of Logarithms

In the properties given next, M and a are positive real numbers, $a \neq 1$, and r is any real number.

The number $\log_a M$ is the exponent to which a must be raised to obtain M. That is,

$$a^{\log_a M} = M \quad \textbf{(1)}$$

The logarithm to the base a of a raised to a power equals that power. That is,

$$\log_a a^r = r \quad \textbf{(2)}$$

Properties of Logarithms

In the following properties, M, N, and a are positive real numbers, $a \neq 1$, and r is any real number.

The Log of a Product Equals the Sum of the Logs

$$\log_a(MN) = \log_a M + \log_a N \quad \textbf{(3)}$$

The Log of a Quotient Equals the Difference of the Logs

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N \quad \textbf{(4)}$$

The Log of a Power Equals the Product of the Power and the Log

$$\log_a M^r = r \log_a M \quad \textbf{(5)}$$

$$a^x = e^{x \ln a} \quad \textbf{(6)}$$

5.5 Properties and Applications of Logarithms

SUMMARY Properties of Logarithms

In the list that follows, a, b, M, N, and r are real numbers. Also, $a > 0$, $a \neq 1$, $b > 0$, $b \neq 1$, $M > 0$, and $N > 0$.

Definition	$y = \log_a x$ means $x = a^y$	
Properties of logarithms	$\log_a 1 = 0$; $\log_a a = 1$	$\log_a M^r = r \log_a M$
	$a^{\log_a M} = M$; $\log_a a^r = r$	$a^x = e^{x \ln a}$
	$\log_a(MN) = \log_a M + \log_a N$	If $M = N$, then $\log_a M = \log_a N$.
	$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$	If $\log_a M = \log_a N$, then $M = N$.
Change-of-Base Formula	$\log_a M = \frac{\log_b M}{\log_b a}$	

Properties of Logarithms (Recall that logs are only defined for positive values of x.)

For the natural logarithm

1. $\ln xy = \ln x + \ln y$
2. $\ln \frac{x}{y} = \ln x - \ln y$
3. $\ln x^y = y \cdot \ln x$
4. $\ln e^x = x$
5. $e^{\ln x} = x$

For logarithms base a

1. $\log_a xy = \log_a x + \log_a y$
2. $\log_a \frac{x}{y} = \log_a x - \log_a y$
3. $\log_a x^y = y \cdot \log_a x$
4. $\log_a a^x = x$
5. $a^{\log_a x} = x$

General Properties

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = 0$
4. $b^{\log_b x} = x$

Useful Identities for Logarithms

For the natural logarithm

1. $\ln e = 1$
2. $\ln 1 = 0$

For logarithms base a

1. $\log_a a = 1$, for all $a > 0$
2. $\log_a 1 = 0$, for all $a > 0$

Natural Logarithms

1. $\ln 1 = 0$
2. $\ln e = 1$
3. $\ln e^{x} = x$
4. $e^{\ln x} = x$

Properties of Natural Logarithms

1. $\ln 1 = 0$ since $e^0 = 1$.
2. $\ln e = 1$ since $e^1 = e$.
3. $\ln e^x = x$ and $e^{\ln x} = x$ inverse property
4. If $\ln x = \ln y$, then $x = y$. one-to-one property

Exponential Laws

$a^0 = 1$, for $a \neq 0$

$x^1 = x$

$x^0 = 1$

$x^{-1} = 1/x$

$\log_a 1 = 0 \qquad \log_a a = 1$

46.

Use the properties of logarithms to expand or simplify the following expression as much as possible. Simplify any numerical expressions that can be evaluated without a calculator.

(a) $\log_9(81x^3)$

(b) $\ln \sqrt[5]{ey}$

(c) $\log_3(\frac{x-4}{x^7})$

(d) $\ln e^{42}$

(e) $\log_{10} 5 + \log_{10} 2$

(f) $e^{\ln 25}$

A	B	C
D	E	F

47.

Let $\log A = 3$ and $\log B = -12$. Find $\log \dfrac{A}{B}$.

48.

Solve the following equations. If there is no solution, state "No Solution".

(a) $\left(\frac{1}{2}\right)^{5x+5} = \left(\frac{1}{4}\right)^{4}$

(b) $3e^{4x} = 90$

(c) $\log_9(x^2 + 12x + 32) - \log_9(x + 8) = 0$

(d) $e^{2x+5} = 12^{\frac{2x}{7}}$

(e) $\log_5(x - 1) + \log_5(x - 3) = 1$

(f) $5^{-x-9} = 625$

(g) $2^{x^2+5x} = 4^{-3}$

49.

Find $f \circ g(x)$ and $g \circ f(x)$ when $f(x) = \ln(x)$ and $g(x) = e^{4x}$.

50.

Find the domain of the function $f(x) = \ln(x-3)$. Determine the range and any asymptotes of $f(x)$.

51.

For $f(x) = 2 + \log(x-5)$.

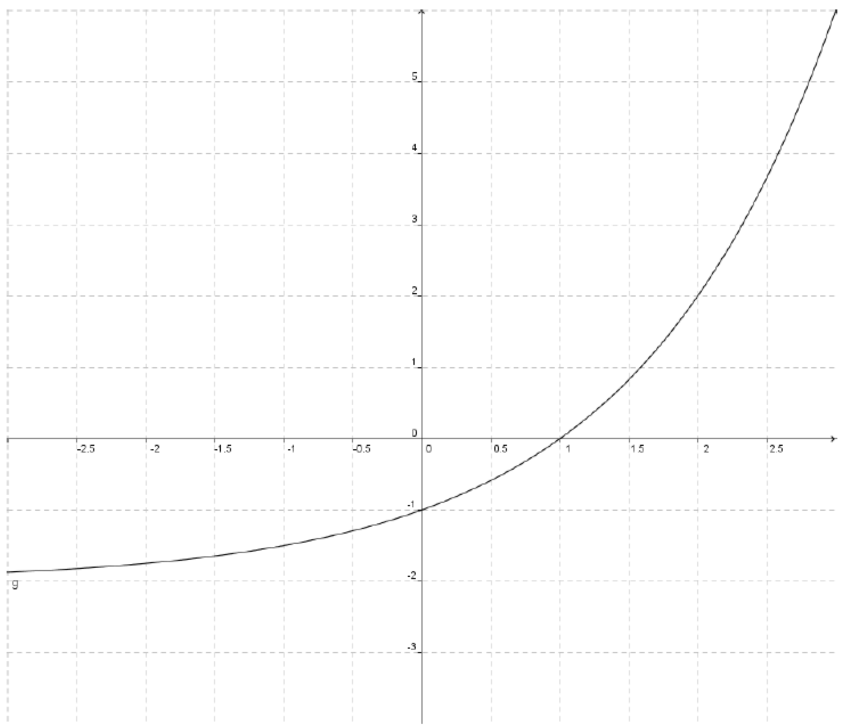
(a) Identify and graph the more basic function that has been shifted, reflected, stretched, or compressed to obtain $f(x)$.

(b) Graph $f(x)$.

52.

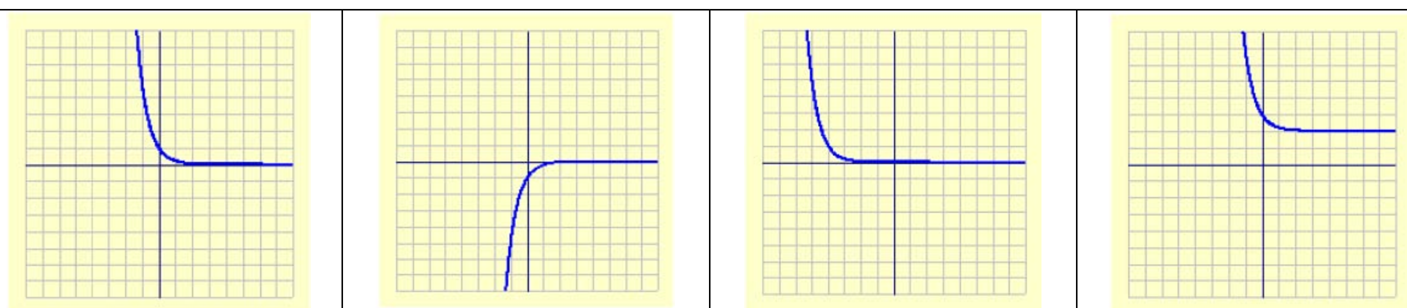
Which function matches the graph shown in the following graph ?

(a) $y = 2^{x+2}$ (b) $y = 2^{x+1} + 2$ (c) $y = 2^{x-2}$ (d) $y = 2^{x} - 2$



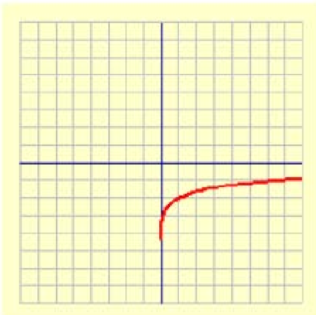
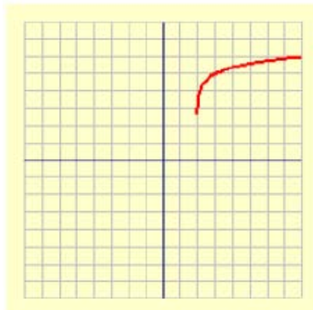
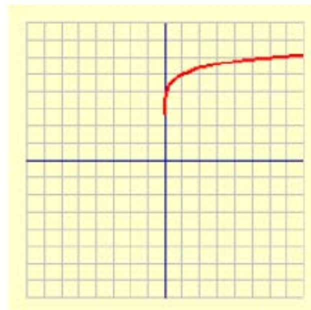
EXTRA PROBLEMS

Indicate which of the following four graphs is the correct graph of the function $q(x) = (\frac{1}{5})^{x}$



(a)	(b)	(c)	(d)

Indicate which of the following four graphs is the correct graph of the function $g(x) = \log_7(x) + 5$

			
(a)	(b)	(c)	(d)