Factoring HW, Ideas and concepts.

First determine if a common monomial factor (Greatest Common Factor) exists. Factor trees may be used to find the GCF of difficult numbers. Be aware of opposites: Ex. (a-b) and (b-a) These may become the same by factoring -1 from one of them.

$$3x - 12 = 3(x - 4)$$
$$x^2y^2 - 3xy^2 = xy^2(x - 3)$$
$$6(x - y) + a(x - y) = (x - y)(6 + a)$$

Provide two new examples from the list of problems given at the end for GCF. Show work.

1	2

If the problem to be factored is a binomial, see if it fits one of the following situations.

A. Difference of two squares:

$$a^2 - b^2 = (a + b)(a - b)$$
$$9x^2 - 25y^2 = (3x + 5y)(3x - 5y)$$
$$(a + b)^2 - 25 = [(a + b) + 5][(a + b) - 5] = (a + b + 5)(a + b - 5)$$

Provide two new examples from the list of problems given at the end for Difference of two squares. Show work.

1	2

B. Sum of two squares:

$a^2 + b^2$ **does not factor (it is prime).**

C. Sum of two cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$8x^3 + 27y^3 = (2x + 3y)(4x^2 - 6xy + 9y^2)$$

Provide two new examples from the list of problems given at the end for Sum of two cubes. Show work.

1	2

Note: Resulting trinomial does not factor.

D. Difference of two cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$
$$x^3 - 64 = (x - 4)(x^2 + 4x + 16)$$

Provide two new examples from the list of problems given at the end for difference of two cubes. Show work.

1	2

Note: Resulting trinomial does not factor.

E. If none of these occur, the binomial does not factor.

3) If the problem is a trinomial, check for one of the following possibilities.

A. Square of a binomial:

$$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$$
$$x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$$
$$4x^2 - 20xy + 25y^2 = (2x - 5y)^2$$

B. If $a = 1$, use reverse foil or trial and error method:

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$
$$x^2 - 7x + 12 = (x - 3)(x - 4)$$
$$x^2 + 3x - 18 = (x + 6)(x - 3)$$
$$x^2 - 3x - 18 = (x - 6)(x + 3)$$

C. If $a \neq 1$, use trial and error method. (Grouping may also be used.)

Provide two new examples from the list of problems given at the end for factoring a trinomial. Show work.

1	2

4) If factoring a polynomial with four terms, possible choices are below.

A. Group first two terms together and last two terms together.

$$5a - 5b - xa + xb = (5a - 5b) + (-xa + xb) = 5(a - b) - x(a - b) = (a - b)(5 - x)$$
$$x^3 - 3x^2 + 2x - 6 = (x^3 - 3x^2) + (2x - 6) = x^2(x - 3) + 2(x - 3) = (x - 3)(x^2 + 2)$$

Provide two new examples from the list of problems given at the end for grouping the first two term. Show work.

1	2

B. Group first three terms together.

$$x^2 + 6x + 9 - y^2 = (x^2 + 6x + 9) - y^2 = (x + 3)^2 - y^2 = [(x + 3) + y][(x + 3) - y] = (x + 3 + y)(x + 3 - y)$$

Provide two new examples from the list of problems given at the end for grouping the first three term. Show work.

1	2

C. Group last three terms together.

$$y^2 - x^2 + 6x - 9 = y^2 - (x^2 - 6x + 9) = y^2 - (x - 3)^2 = [y + (x - 3)][y - (x - 3)] = (y + x - 3)(y - x + 3)$$

Provide two new examples from the list of problems given at the end for grouping the last three terms Show work.

1	2

BE SURE YOUR ANSWERS WILL NOT FACTOR FURTHER!
All answers may be checked by multiplication.

PRACTICE PROBLEMS:

1. $y^3 + 9y^2$
2. $5x^2y^3 + 15x^3y^2$
3. $12t^5 - 20t^4 + 8t^2 - 16$
4. $p^2 - 36$
5. $25 - x^2$
6. $4a^3 - 49a$
7. $(a+b)^2 - 100$
8. $9 - (x-y)^2$
9. $y^3 + 8$
10. $64y^4 + y$
11. $x^3 - 27$
12. $5x^3 - 40y^3$
13. $2y^4 - 128y$
14. $t^6 - 64$
15. $x^2 - 10x + 25$
16. $4a^2 + 16a + 16$
17. $16y^2 + 56y + 49$
18. $-20xy + 4y^2 + 25x^2$
19. $x^2 + 9x + 20$
20. $2y^2 - 16y + 32$
21. $3x + x^2 - 10$
22. $y^2 + 5y - 84$
23. $8x^2 - 16 - 28x$
24. $12x^3 - 31x^2 + 20x$
25. $6a^2 - 7a - 10$
26. $8 - 6x - 9x^2$
27. $6x^6 + x^3 - 2$
28. $2x^8 - 14x^4 + 20$
29. $y^3 - y^2 + 2y - 2$
30. $x^4 - x^3 - x + x^2$
31. $x^3 + 8x^2 - x - 8$
32. $p^2q - 25q + 3p^2 - 75$
33. $16 - x^2 + 2xy - y^2$
34. $2xy - x^2y - 6 + 3x$
35. $6x^2 + 23x + 20$
36. $9x^2 + 15x + 4$
37. $8m^2 - 6m - 9$
38. $25 - 10x + x^2$
39. $16 - w^4$
40. $ay - yx - x^2 + ax$

ANSWERS:

1. $y^2(y+9)$ 2. $5x^2y^2(y+3x)$ 3. $4(3t^5 - 5t^4 + 2t^2 - 4)$ 4. $(p+6)(p-6)$
5. $(5+x)(5-x)$ 6. $a(2a+7)(2a-7)$ 7. $(a+b+10)(a+b-10)$
8. $(3+x-y)(3-x+y)$ 9. $(y+2)(y^2-2y+4)$ 10. $y(4y+1)(16y^2-4y+1)$
11. $(x-3)(x^2+3x+9)$ 12. $5(x-2y)(x^2+2xy+4y^2)$ 13. $2y(y-4)(y^2+4y+16)$
14. $(t+2)(t^2-2t+4)(t-2)(t^2+2t+4)$ 15. $(x-5)^2$ 16. $4(a+2)^2$ 17. $(4y+7)^2$
18. $(5x-2y)^2$ 19. $(x+5)(x+4)$ 20. $2(y-4)^2$ 21. $(x+5)(x-2)$ 22. $(y+12)(y-7)$
23. $4(2x+1)(x-4)$ 24. $x(4x-5)(3x-4)$ 25. $(a-2)(6a+5)$ 26. $(4+3x)(2-3x)$
27. $(3x^3+2)(2x^3-1)$ 28. $2(x^4-5)(x^4-2)$ 29. $(y-1)(y^2+2)$ 30. $x(x^2+1)(x-1)$
31. $(x+8)(x+1)(x-1)$ 32. $(q+3)(p+5)(p-5)$ 33. $(4+x-y)(4-x+y)$
34. $(2-x)(xy-3)$ 35. $(3x+4)(2x+5)$ 36. $(3x+1)(3x+4)$ 37. $(4m+3)(2m-3)$
38. $(5-x)^2$ or $(x-5)^2$ 39. $(4+w^2)(2+w)(2-w)$ 40. $(y+x)(a-x)$

MORE PRACTICE PROBLEMS:

41. $x^2 - 6x - 16$
42. $x^2 - 10xy + 24y^2$
43. $x^2 + 3x + 2$
44. $x^2 - 3x + 2$
45. $x^2 - x - 30$
46. $x^2 + 7x - 8$
47. $x^2 + x - 2$
48. $x^2 - 5xy + 6y^2$
49. $x^2 + 10x + 16$
50. $x^2 + x - 72$
51. $x^2 - 8x - 9$
52. $x^2 + 2x - 48$
53. $x^2 - 13xy + 42y^2$
54. $x^2 + 8x + 12$
55. $4x^3 - 8x^2 - 12x$
56. $2x^3 - 2x^2 - 4x$
57. $2x^3 - 4x^2 - 6x$
58. $3x^3 - 6x^2 - 9x$
59. $5x^3y - 35x^2y + 50xy$
60. $3x^3y + 18x^2y - 21xy$
61. $4x^2 + 1 - 4x$
62. $15x^2 + 12 + 29x$
63. $8r^2 - 2r - 3$
64. $35a^2 + 3a - 20$
65. $25x^2 + 8 + 30x$
66. $12x^2 + 3 + 13x$
67. $9x^2 - 27xy + 20y^2$
68. $25u^2 - 15u - 18$
69. $12f^2 - 4f - 5$
70. $5z^2 + 3z + 4$
71. $4x^2 + 15 + 16x$
72. $20x^2 + 6 + 23x$
73. $6x^2 - 19xy + 10y^2$
74. $35p^2 + 13p - 4$
75. $50x^2 + 10x - 12$
76. $-30x^2 - 25x + 30$
77. $-18x^2 + 18x + 20$
78. $3x^3 - 22x^2 + 7x$
79. $15x^2 - 18x - 24$
80. $4x^3 - 25x^2 + 6x$

ANSWERS:

41. $(x-8)(x+2)$ 42. $(x-6y)(x-4y)$ 43. $(x+2)(x+1)$ 44. $(x-2)(x-1)$
45. $(x-6)(x+5)$ 46. $(x+8)(x-1)$ 47. $(x+2)(x-1)$ 48. $(x-3y)(x-2y)$
49. $(x+8)(x+2)$ 50. $(x+9)(x-8)$ 51. $(x-9)(x+1)$ 52. $(x+8)(x-6)$
53. $(x-7y)(x-6y)$ 54. $(x+6)(x+2)$ 55. $4x(x-3)(x+1)$ 56. $2x(x-2)(x+1)$
57. $2x(x-3)(x+1)$ 58. $3x(x-3)(x+1)$ 59. $5xy(x-5)(x-2)$ 60. $3xy(x+7)(x-1)$ 61. $(2x-1)^2$ 62. $(3x+4)(5x+3)$ 63. $(2r+1)(4r-3)$ 64. $(5a+4)(7a-5)$
65. $(5x+4)(5x+2)$ 66. $(3x+1)(4x+3)$ 67. $(3x-5y)(3x-4y)$ 68. $(5u+3)(5u-6)$ 69. $(2f+1)(6f-5)$ 70. $Prime\ (Cannot\ be\ factored)$ 71. $(2x+3)(2x+5)$
72. $(5x+2)(4x+3)$ 73. $(2x-5y)(3x-2y)$ 74. $(7p+4)(5p-1)$
75. $2(5x+3)(5x-2)$ 76. $-5(2x+3)(3x-2)$ 77. $-2(3x-5)(3x+2)$
78. $x(3x-1)(x-7)$ 79. $3(5x+4)(x-2)$ 80. $x(4x-1)(x-6)$

MORE PRACTICE PROBLEMS:

81. $125x^3 - 1$
82. $w^2 - 64$
83. $y^2 - 12y + 36$
84. $x^2 - 8x - 48$
85. $a^3 - 7a^2 + 12a$
86. $25a^2 + 8b^2$
87. $(x-3)(x+7) + (x-3)(x-4)$
88. $6x^2 + 12x + 6$
89. $y^2 - 11y + 18$
90. $40 + 3b - b^2$
91. $3x^5 - 12x^2$
92. $250x^3 + 2$
93. $7xy^4 - 7xz^4$
94. $2y^4 + 5y^3 - 12y^2$
95. $24x^2 - 7x - 5$
96. $y^2 + 14y - 32$
97. $0.04w^2 + 0.28w + 0.49$
98. $4x^3 + 40x^2 + 64x$
99. $64y^3 + 27$
100. $\frac{1}{81} - x^2$
101. $5x^2 - 2x + 3$
102. $x^3 - 343$
103. $40y^2 + 28y - 48$
104. $3ab - 5bc + bd$
105. $8c^6 - 125d^6$
106. $81 - 18z + z^2$
107. $x^4 + 10x^3 + 25x^2$
108. $xz - xw - yz + yw$
109. $y^2 + 5y - 36$
110. $x^2 - 11x - 42$
111. $7a^2 - 7b^2$
112. $216 - a^3$
113. $81 + 18y + y^2$
114. $b^2 - 5b - 14$
115. $q^4 - 10q^3 + 21q^2$
116. $9x^2y^2 - 25y^4$
117. $105 + 8x - x^2$
118. $x^2 - 3x - 2$
119. $6y^3 + 48$
120. $a^3 - 14a^2 + 49a$

ANSWERS:

81. $(5x-1)(25x^2+5x+1)$ 82. $(w+8)(w-8)$ 83. $(y-6)^2$ 84. $(x-12)(x+4)$
85. $a(a-4)(a-3)$ 86. *Prime (Cannot be factored)* 87. $(x-3)(2x+3)$
88. $6(x+1)^2$ 89. $(y-9)(y-2)$ 90. $(8-b)(5+b)$ 91. $3x^2(x^3-4)$
92. $2(5x+1)(25x^2-5x+1)$ 93. $7x(y^2+z^2)(y+z)(y-z)$ 94. $y^2(2y-3)(y+4)$
95. $(8x-5)(3x+1)$ 96. $(y-2)(y+16)$ 97. $(0.2w+0.7)^2$ 98. $4x(x+2)(x+8)$
99. $(4y+3)(16y^2-12y+9)$ 100. $\left(\frac{1}{9}+x\right)\left(\frac{1}{9}-x\right)$ 101. *Prime (Cannot be factored)* 102. $(x-7)(x^2+7x+49)$ 103. $4(2y+3)(5y-4)$ 104. $b(3a-5c+d)$
105. $(2c^2-5d^2)(4c^4+10c^2d^2+25d^4)$ 106. $(9-z)^2$ 107. $x^2(x+5)^2$
108. $(x-y)(z-w)$ 109. $(y-4)(y+9)$ 110. $(x-14)(x+3)$ 111. $7(a+b)(a-b)$
112. $(6-a)(36+6a+a^2)$ 113. $(9+y)^2$ 114. $(b-7)(b+2)$ 115. $q^2(q-3)(q-7)$ 116. $y^2(3x+5y)(3x-5y)$ 117. $(7+x)(15-x)$ 118. *Prime (Cannot be factored)*
119. $6(y+2)(y^2-2y+4)$ 120. $a(a-7)^2$

121. $3y^2 - 34y - 24$

122. $a^2 + 8a + 16$

123. $y^2 - 121$

124. $42 + a - a^2$

125. $9x^3 - 24x^2 + 16x$

126. $x^3 - \frac{1}{8}$

127. $10w^2 + 29w - 21$

128. $16x^2 + 54x - 7$

129. $27x^2 - 30x - 8$

130. $x^6 - 1$

131. $x^2 - 0.6x + 0.09$

132. $4x^2 - 13x - 35$

133. $125x^6 - 81$

134. $49x^3 - 14x^2 + x$

135. $40y^2 + 7y - 3$

136. $15w^2 - 15w - 90$

137. $0.04a^2 - 0.49b^2$

138. $x^3y^2 + 7x^2y^2 - 18xy^2$

139. $2x^6 - 54y^6$

140. $\frac{1}{4}x^2 - 5x + 25$

ANSWERS:

121. $(y - 12)(3y + 2)$ 122. $(a + 4)^2$

123. $(y + 11)(y - 11)$ 124. $(7 - a)(6 + a)$ 125. $x(3x - 4)^2$ 126. $\left(x - \frac{1}{2}\right)\left(x^2 + \frac{1}{2}x + \frac{1}{4}\right)$

127. $(5w - 3)(2w + 7)$ 128. $(2x + 7)(8x - 1)$ 129. $(9x + 2)(3x - 4)$

130. $(x + 1)(x - 1)(x^2 - x + 1)(x^2 + x + 1)$ 131. $(x - 0.3)^2$ 132. $(x - 5)(4x + 7)$

133. *Prime (Cannot be factored)* 134. $x(7x - 1)^2$ 135. $(8y + 3)(5y - 1)$

136. $15(w + 2)(w - 3)$ 137. $(0.2a + 0.7b)(0.2a - 0.7b)$ 138. $xy^2(x - 2)(x + 9)$

139. $2(x^2 - 3y^2)(x^4 + 3x^2y^2 + 9y^4)$ 140. $\left(\frac{1}{2}x - 5\right)^2$