## **Properties of Exponents and Logarithms**

## **Exponents**

Let a and b be real numbers and m and n be integers. Then the following properties of exponents hold, provided that all of the expressions appearing in a particular equation are defined.

1. 
$$a^{m}a^{n} = a^{m+n}$$
  
2.  $(a^{m})^{n} = a^{mn}$   
3.  $(ab)^{m} = a^{m}b^{m}$   
4.  $\frac{a^{m}}{a^{n}} = a^{m-n}, a \neq 0$   
5.  $\left(\frac{a}{b}\right)^{m} = \frac{a^{m}}{b^{m}}, b \neq 0$   
6.  $a^{-m} = \frac{1}{a^{m}}, a \neq 0$   
7.  $a^{\frac{1}{n}} = \sqrt[n]{a}$   
8.  $a^{0} = 1, a \neq 0$   
9.  $a^{\frac{m}{n}} = \sqrt[n]{a^{m}} = \left(\sqrt[n]{a}\right)^{m}$ 

where m and n are integers in properties 7 and 9.

## Logarithms

Definition:  $y = \log_a x$  if and only if  $x = a^y$ , where a > 0. In other words, logarithms are exponents.

Remarks:

- $\log x$  always refers to log base 10, i.e.,  $\log x = \log_{10} x$ .
- $\ln x$  is called the natural logarithm and is used to represent  $\log_e x$ , where the irrational number  $e \approx 2.71828$ . Therefore,  $\ln x = y$  if and only if  $e^y = x$ .
- Most calculators can directly compute logs base 10 and the natural log. For any other base it is necessary to use the change of base formula:  $\log_b a = \frac{\ln a}{\ln b}$  or  $\frac{\log_{10} a}{\log_{10} b}$ .

**Properties of Logarithms** (Recall that logs are only defined for positive values of x.)

For the natural logarithmFor logarithms base a1.  $\ln xy = \ln x + \ln y$ 1.  $\log_a xy = \log_a x + \log_a y$ 2.  $\ln \frac{x}{y} = \ln x - \ln y$ 2.  $\log_a \frac{x}{y} = \log_a x - \log_a y$ 3.  $\ln x^y = y \cdot \ln x$ 3.  $\log_a x^y = y \cdot \log_a x$ 4.  $\ln e^x = x$ 4.  $\log_a a^x = x$ 5.  $e^{\ln x} = x$ 5.  $a^{\log_a x} = x$ 

## Useful Identities for Logarithms

For the natural logarithm	For logarithms base $a$
1. $\ln e = 1$	1. $\log_a a = 1$ , for all $a > 0$
2. $\ln 1 = 0$	2. $\log_a 1 = 0$ , for all $a > 0$