## Properties of Exponents and Logarithms

## Exponents

Let $a$ and $b$ be real numbers and $m$ and $n$ be integers. Then the following properties of exponents hold, provided that all of the expressions appearing in a particular equation are defined.

1. $a^{m} a^{n}=a^{m+n}$
2. $\left(a^{m}\right)^{n}=a^{m n}$
3. $(a b)^{m}=a^{m} b^{m}$
4. $\frac{a^{m}}{a^{n}}=a^{m-n}, a \neq 0$
5. $\left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}, b \neq 0$
6. $a^{-m}=\frac{1}{a^{m}}, a \neq 0$
7. $a^{\frac{1}{n}}=\sqrt[n]{a}$
8. $a^{0}=1, a \neq 0$
9. $a^{\frac{m}{n}}=\sqrt[n]{a^{m}}=(\sqrt[n]{a})^{m}$
where $m$ and $n$ are integers in properties 7 and 9 .

## Logarithms

Definition: $y=\log _{a} x$ if and only if $x=a^{y}$, where $a>0$.
In other words, logarithms are exponents.

## Remarks:

- $\log x$ always refers to $\log$ base 10 , i.e., $\log x=\log _{10} x$.
- $\ln x$ is called the natural logarithm and is used to represent $\log _{e} x$, where the irrational number $e \approx 2.71828$. Therefore, $\ln x=y$ if and only if $e^{y}=x$.
- Most calculators can directly compute logs base 10 and the natural log. For any other base it is necessary to use the change of base formula: $\log _{b} a=\frac{\ln a}{\ln b}$ or $\frac{\log _{10} a}{\log _{10} b}$.
Properties of Logarithms (Recall that logs are only defined for positive values of $x$.)
For the natural logarithm For logarithms base $a$

1. $\ln x y=\ln x+\ln y$
2. $\log _{a} x y=\log _{a} x+\log _{a} y$
3. $\ln \frac{x}{y}=\ln x-\ln y$
4. $\log _{a} \frac{x}{y}=\log _{a} x-\log _{a} y$
5. $\ln x^{y}=y \cdot \ln x$
6. $\log _{a} x^{y}=y \cdot \log _{a} x$
7. $\ln e^{x}=x$
8. $\log _{a} a^{x}=x$
9. $e^{\ln x}=x$
10. $a^{\log _{a} x}=x$

## Useful Identities for Logarithms

For the natural logarithm For logarithms base $a$

1. $\ln e=1$
2. $\log _{a} a=1$, for all $a>0$
3. $\ln 1=0$
4. $\log _{a} 1=0$, for all $a>0$
