Exponents [TE1-B]

(i)	$a^m a^n = a^{m+n}$	e.g.	$a^2 a^3 = (aa)(aaa) = a^5$
(ii)	$(a^m)^n = a^{mn}$	e.g.	$(a^2)^3 = (aa)(aa)(aa) = a^6$
(iii)	$\dfrac{a^m}{a^n} = a^{m-n}$	e.g.	$\dfrac{a^2}{a^5} = a^{-3} = \dfrac{1}{a^3}$
(iv)	$(ab)^m = a^m b^m$	e.g.	$(ab)^2 = (ab)(ab) = (aa)(bb) = a^2 b^2$
(v)	$\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$	e.g.	$\left(\dfrac{a}{b}\right)^2 = \dfrac{a}{b} \cdot \dfrac{a}{b} = \dfrac{aa}{bb} = \dfrac{a^2}{b^2}$

Properties of Exponents

Throughout this table, a and b may be taken to represent constants, variables, or more complicated algebraic expressions. The letters n and m represent integers.

	Property	Example
1.	$a^n \cdot a^m = a^{n+m}$	$(-3)^3 \cdot (-3)^{-1} = (-3)^{3+(-1)} = (-3)^2 = 9$
2.	$\dfrac{a^n}{a^m} = a^{n-m}$	$\dfrac{7^9}{7^{10}} = 7^{9-10} = 7^{-1}$
3.	$a^{-n} = \dfrac{1}{a^n}$	$5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$ and $x^3 = \dfrac{1}{x^{-3}}$
4.	$(a^n)^m = a^{nm}$	$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$

Properties of Exponents, cont.

	Property	Example
5.	$(ab)^n = a^n b^n$	$(7x)^3 = 7^3 x^3 = 343x^3$ and $\left(-2x^5\right)^2 = (-2)^2 (x^5)^2 = 4x^{10}$
6.	$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$\left(\dfrac{3}{x}\right)^2 = \dfrac{3^2}{x^2} = \dfrac{9}{x^2}$ and $\left(\dfrac{1}{3z}\right)^2 = \dfrac{1^2}{(3z)^2} = \dfrac{1}{9z^2}$
7.	$\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n}$	$\left(\dfrac{5}{4}\right)^{-3} = \dfrac{4^3}{5^3} = \dfrac{64}{125}$
8.	$\dfrac{a^{-m}}{b^{-n}} = \dfrac{b^n}{a^m}$	$\dfrac{3^{-2}}{2^{-4}} = \dfrac{2^4}{3^2} = \dfrac{16}{9}$

In the above table, it is assumed that every expression is defined. That is, if an exponent is 0, then the base is non-zero, and if an expression appears in the denominator of a fraction, then that expression is non-zero. Remember that $a^0 = 1$ for every $a \neq 0$.

1. $a^m a^n = a^{m+n}$	5. $\frac{a^m}{a^n} = a^{m-n}$, for $a \neq 0$
2. $(a^m)^n = a^{mn}$	6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, for $b \neq 0$
3. $(ab)^m = a^m b^m$	7. $a^{-m} = \frac{1}{a^m}$, for $a \neq 0$
4. $a^0 = 1$, for $a \neq 0$	

Let a and b be real numbers and m and n be integers. Then the following properties of exponents hold, provided that all of the expressions appearing in a particular equation are defined.

1. $a^m a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $(ab)^m = a^m b^m$
4. $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$
5. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$
6. $a^{-m} = \frac{1}{a^m}$, $a \neq 0$
7. $a^{\frac{1}{n}} = \sqrt[n]{a}$
8. $a^0 = 1$, $a \neq 0$
9. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

where m and n are integers in properties 7 and 9.

Name the base and exponent in the following expressions. Then, use the definition of exponents as repeated multiplication to simplify.

8^2 The base is _8_ The exponent is _2_ $8^2 =$ $\underline{(8)(8) = 64}$	$(-12)^2$ The base is $\underline{-12}$ The exponent is _2_ $(-12)^2 =$ $\underline{(-12)(-12) = 144}$
$(-10)^2$ The base is $\underline{-10}$ The exponent is _2_ $(-10)^2 =$ $\underline{(-10)(-10) = 100}$	$(-2)^3$ The base is $\underline{-2}$ The exponent is _3_ $(-2)^3 =$ $\underline{(-2)(-2)(-2) = -8}$
$\left(\frac{3}{5}\right)^2$ The base is $\frac{3}{5}$ The exponent is _2_ $\left(\frac{3}{5}\right)^2 = \left(\frac{3}{5}\right)\left(\frac{3}{5}\right) = \frac{9}{25}$	$\left(-\frac{3}{4}\right)^2$ The base is $-\frac{3}{4}$ The exponent is _2_ $\left(-\frac{3}{4}\right)^2 = \frac{9}{16}$

Adding Exponents & Polynomials [See adding polynomials]

Subtracting Exponents & Polynomials [See subtracting Polynomials]

Multiplying [TE1-B]

$v^4 \cdot v^3 = \underline{v^{4+3} = v^7}$	$y^5 \cdot y^8 \cdot y^{11} \cdot y^{14} = \underline{y^{5+8+11+14} = y^{38}}$
$(b^2)^2 = \underline{b^{2\cdot 2} = b^4}$	$(-6u)^3 = \underline{(-6)^3(u^3) = -6^3u^3 \text{ or } -216u^3}$
$(-3ab)^3 = \underline{(-3)^3(a)^3(b^3) = -27a^3b^3}$	$(4p^6)^5 =$ $\underline{(4)^5 \cdot (p^6)^5 = 1024 \cdot p^{6\cdot 5} = 1024p^{30}}$
$(t^9)^3(t^5)^7 = \underline{(t^{9\cdot 3})(t^{5\cdot 7})}$ $\underline{= (t^{27})(t^{35})}$ $\underline{= t^{27+35}}$ $\underline{= t^{62}}$	$(2r^2)^7(2r)^9 = \underline{(2^7r^{2\cdot 7})(2^9r^9)}$ $\underline{= (128r^{14})(512r^9)}$ $\underline{= (128 \cdot 512)r^{14+9}}$ $\underline{= 65{,}536r^{23}}$
$\left(\frac{4}{5}y^7p^3\right)^4 = \left(\frac{4}{5}\right)^4 y^{7\cdot 4}p^{3\cdot 4} = \frac{256}{625}y^{28}p^{12}$	

Dividing [TE2-B]

Division	Negative Exponents
$\frac{7^9}{7^{11}} = 7^{9-11} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$	$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$
$\frac{9^3}{9^0} = \underline{9^{3-0} = 9^3 = 729}$	$(3a)^{-1} = \frac{1}{3a}$

$\left(\frac{x}{6}\right)^4 = \frac{x^4}{6^4} = \frac{x^4}{1{,}296}$ $\left(\frac{7}{y}\right)^3 = \frac{7^3}{y^3} = \frac{343}{y^3}$ $(-4b^2)^0 = \underline{\ 1\ }$ $(-64z^3)^1 = \underline{-64z^3}$	$\frac{(2r)^{-12}}{(2r)^{-5}} = (2r)^{-12-(-5)}$ $= (2r)^{-7}$ $= \frac{1}{(2r)^7}$ $= \frac{1}{2^7 r^7}$ $= \frac{1}{128r^7}$
$(2cp)^9 =$ $\underline{2^9 \cdot c^9 \cdot p^9 = 512c^9p^9}$	$t^3 \cdot t^{-7} = t^{3+(-7)} = t^{-4} = \frac{1}{t^4}$
$(7s^9v)^1 = \underline{7^1 \cdot (s^9)^1 \cdot v^1 = 7s^9v}$	$\frac{q^{13}}{(q^4)^6} = \frac{q^{13}}{q^{4\cdot 6}} = \frac{q^{13}}{q^{24}} = q^{13-24} = q^{-11} = \frac{1}{q^{11}}$