EXAM 1 CHAPTER 1

1.1.a THE REAL NUMBER SYSTEM

You were asked to write the following in interval notation:

$$\{\ x \mid -11 < x \leq 10\ \}$$

By definition of interval notation, $\{ x \mid a < x \leq b \}$ is all real numbers between a and b, including b, but not a. Therefore your answer should have the form $(a, b]$.

Interval Notation

Notation	**Meaning**
(a, b)	$\{ x \mid a < x < b \}$, or all real numbers strictly between a and b.
$[a, b]$	$\{ x \mid a \leq x \leq b \}$, or all real numbers strictly between a and b, including both a and b.
$(a, b]$	$\{ x \mid a < x \leq b \}$, or all real numbers strictly between a and b, including b but not a.
$(-\infty, b)$	$\{ x \mid x < b \}$, or all real numbers less than b.
$[a, \infty)$	$\{ x \mid x \geq a \}$, or all real numbers greater than or equal to a.

In interval notation, the left endpoint is always written first. Intervals of the form (a, b) are called **open** intervals, while those of the form $[a, b]$ are **closed** intervals. The interval $(a, b]$ is **half-open** (or **half-closed**). Of course, a half-open interval may be open at either endpoint, as long as it is closed at the other. The symbols $-\infty$ and ∞ indicate that the interval extends indefinitely in, respectively, the left and the right directions. Note that $(-\infty, b)$ excludes the endpoint b, while $[a, \infty)$ includes the endpoint a.

You were asked to write the following in interval notation:

$$\{\ x \mid -11 < x \leq 10\ \}$$

Because $-11 < x$ is a strict inequality, the endpoint -11 is not included in the interval. So it is necessary to use a parenthesis by the number -11.

Since $x \leq 10$ is a non-strict inequality, the endpoint 10 is included in the interval. So it is necessary to use a bracket by the number 10.

Since -11 is to the left of 10 on the number line, -11 is written first in the interval notation.

Answer: $(-11, \ 10]$

Write the first four terms of the set described by

$$\left\{ \frac{x}{x+6} \mid x \text{ is a natural number} \right\}$$

You were asked to write the first four terms of the set described by the following:

$$\frac{x}{x+6}$$

First we will write the first four terms of the set of x. Since the set of x is the set of natural numbers, the first four terms are:

1, 2, 3, 4

Now we perform the operation $\frac{x}{x+6}$ to each of these terms. So we have:

$$\frac{1}{1+6} = \frac{1}{7},\ \frac{2}{2+6} = \frac{2}{8} = \frac{1}{4},\ \frac{3}{3+6} = \frac{3}{9} = \frac{1}{3},\ \frac{4}{4+6} = \frac{4}{10} = \frac{2}{5}$$

Answer : $\frac{1}{7}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{5}$

1.1b

You were asked to evaluate the following expression, using the correct order of operations:

$$-5 - 3 \cdot -5 \div 2 + (-4)^3$$

Because there are no grouping symbols in this expression, the first step is to calculate exponents and roots.

$$-5 - 3 \cdot -5 \div 2 + (-4)^3 = -5 - 3 \cdot -5 \div 2 - 64$$ Simplify the term with an exponent.

$$= -5 + \frac{15}{2} - 64$$ Perform multiplications and divisions from left to right.

$$= -\frac{10}{2} + \frac{15}{2} - \frac{128}{2}$$ Find a common denominator.

$$= \frac{-123}{2}$$ Add.

Answer: $\frac{-123}{2}$

You were asked to evaluate the following expression for the given values of the variables:

$$|\ x + 5y\ | + (\ 5z + 1\ ) \text{ for } x = -1,\ y = 5 \text{ and } z = -2$$

Begin by replacing all instances of *x* with –1, all instances of *y* with 5, and all instances of *z* with –2, as follows:

$$|\ x + 5y\ | + (\ 5z + 1\ ) = |\ (\ -1\ ) + 5(\ 5\ )\ | + (\ 5(\ -2\ ) + 1\ )$$

Insert –1 for *x*, 5 for *y*, and –2 for *z*.

$$= |\ -1 + (\ 25\ )\ | + (\ -10 + 1\ )$$

$$= |\ 24\ | + (\ -9\ )$$

$$= 24 - 9$$

$$= 15$$

Calculate (following the rules for the order of operations) and simplify.

Order of Operations

1. If the expression is a fraction, simplify the numerator and denominator individually, according to the guidelines in the following steps.
2. **Parentheses, braces, and brackets** are all used as grouping symbols. Simplify expressions within each set of grouping symbols, if any are present, working from the innermost outward.
3. Simplify all **powers (exponents) and roots**.
4. Perform all **multiplications and divisions** in the expression in the order they occur, working from left to right.
5. Perform all **additions and subtractions** in the expression in the order they occur, working from left to right.

You were asked to evaluate the following expression for the given values of the variables:

$$-5\sqrt{x + 1} + 5y^2 \text{ for } x = 35 \text{ and } y = -1$$

Begin by replacing all instances of *x* with 35 and all instances of *y* with –1, as follows:

$$-5\sqrt{x + 1} + 5y^2 = -5\sqrt{(\ 35\ ) + 1} + 5(\ -1\ )^2$$

Insert 35 for *x* and –1 for *y*.

$$= -5\sqrt{36} + 5(\ 1\ )$$

$$= -5(\ 6\ ) + 5$$

$$= -30 + 5$$

$$= -25$$

Calculate (following the rules for the order of operations) and simplify.

Answer: –25

1.2b

	Property	Example
1.	$a^n \cdot a^m = a^{n+m}$	$(-3)^3 \cdot (-3)^{-1} = (-3)^{3+(-1)} = (-3)^2 = 9$
2.	$\dfrac{a^n}{a^m} = a^{n-m}$	$\dfrac{7^9}{7^{10}} = 7^{9-10} = 7^{-1}$
3.	$a^{-n} = \dfrac{1}{a^n}$	$5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$ and $x^3 = \dfrac{1}{x^{-3}}$
4.	$(a^n)^m = a^{nm}$	$(2^3)^2 = 2^{3\cdot 2} = 2^6 = 64$
5.	$(ab)^n = a^n b^n$	$(7x)^3 = 7^3x^3 = 343x^3$ and $(-2x^5)^2 = (-2)^2(x^5)^2 = 4x^{10}$
6.	$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$\left(\dfrac{3}{x}\right)^2 = \dfrac{3^2}{x^2} = \dfrac{9}{x^2}$ and $\left(\dfrac{1}{3z}\right)^2 = \dfrac{1^2}{(3z)^2} = \dfrac{1}{9z^2}$
7.	$\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n}$	$\left(\dfrac{5}{4}\right)^{-3} = \dfrac{4^3}{5^3} = \dfrac{64}{125}$
8.	$\dfrac{a^{-m}}{b^{-n}} = \dfrac{b^n}{a^m}$	$\dfrac{3^{-2}}{2^{-4}} = \dfrac{2^4}{3^2} = \dfrac{16}{9}$

In the above table, it is assumed that every expression is defined. That is, if an exponent is 0, then the base is non-zero, and if an expression appears in the denominator of a fraction, then that expression is non-zero. Remember that $a^0 = 1$ for every $a \neq 0$.

You were asked to simplify the following expression, writing your answer with only positive exponents.

$$\frac{1}{-3y^{-6}}$$

Before trying to simplify this expression, it might be helpful to review the properties of exponents.

So, this expression can be simplified as follows:

$$\frac{1}{-3y^{-6}} = \frac{1}{-3} \cdot \frac{1}{y^{-6}}$$

$$= \frac{1}{-3} \cdot \frac{1}{\frac{1}{y^6}} \qquad a^{-n} = \frac{1}{a^n}$$

$$= \frac{1}{-3} \cdot y^6$$

$$= \frac{-y^6}{3}$$

Answer: $\dfrac{-y^6}{3}$

You were asked to use the properties of exponents to simplify the following expression, writing your answer with only positive exponents.

$$((4x^{-4}z^{2})^{4})^{-1}$$

Before trying to simplify this expression, it might be helpful to review the properties of exponents.

So, this expression can be simplified as follows:

$$((4x^{-4}z^{2})^{4})^{-1} = \frac{1}{(4x^{-4}z^{2})^{4}} \qquad a^{-n} = \frac{1}{a^{n}}$$

$$= \frac{1}{4^{4}} \cdot \frac{1}{x^{-4\cdot 4}} \cdot \frac{1}{z^{2\cdot 4}} \qquad (a^{n})^{m} = a^{n\cdot m}$$

$$= \frac{1}{256} \cdot \frac{1}{\frac{1}{x^{16}}} \cdot \frac{1}{z^{8}} \qquad a^{-n} = \frac{1}{a^{n}}$$

$$= \frac{x^{16}}{256z^{8}}$$

You were asked to simplify the following expression, writing your answer with only positive exponents.

$$\frac{s^{11}z^{-3}}{s^{4}z^{6}}$$

Before trying to simplify this expression, it might be helpful to review the properties of exponents.

So, this expression can be simplified as follows:

$$\frac{s^{11}z^{-3}}{s^{4}z^{6}} = \frac{s^{11}}{s^{4}} \cdot \frac{z^{-3}}{z^{6}}$$

$$= s^{11-4} \cdot z^{-3-6} \qquad \frac{a^{n}}{a^{m}} = a^{n-m}$$

$$= s^{7} \cdot z^{-9}$$

$$= \frac{s^{7}}{z^{9}} \qquad a^{-n} = \frac{1}{a^{n}}$$

Answer: $\frac{s^{7}}{z^{9}}$

1.2c

Properties of Radicals

Throughout the following table, a and b may be taken to represent constants, variables, or more complicated algebraic experessions. the letters n and m represent natural numbers. It is assumed that all expressions are defined and are real numbers.

	Property	Example
1.	$\sqrt[n]{a^n} = a$ if n is odd	$\sqrt[3]{(-5)^3} = -5$, $\sqrt[7]{3^7} = 3$
2.	$\sqrt[n]{a^n} = \lvert a \rvert$ if n is even	$\sqrt[4]{(-6)^4} = \lvert -6 \rvert = 6$
3.	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{3x^6y^2} = \sqrt[3]{3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{y^2} = \sqrt[3]{3} \cdot x \cdot x \cdot \sqrt[3]{y^2} = x^2 \cdot \sqrt[3]{3y^2}$
4.	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\sqrt[4]{\frac{x^4}{16}} = \frac{\sqrt[4]{x^4}}{\sqrt[4]{16}} = \frac{\lvert x \rvert}{2}$
5.	$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt[3]{\sqrt{64}} = \sqrt[3]{\sqrt[2]{64}} = \sqrt[6]{64} = 2$

You were asked to simplify the following radical expression, assuming all variables are positive.

$$\sqrt{25\,y^2}$$

Before simplifying this radical expression, it may be helpful to review the Properties of Radicals.

At this point, this expression can be simplified as follows:

$$\sqrt{25\,y^2} = \sqrt{25} \cdot \sqrt{y^2}$$

$$= 5 \cdot y$$

Since we are given that all variables are positive, no absolute value bars are necessary.

$$= 5y$$

You were asked to simplify the following radical expression by rationalizing the denominator.

$$\frac{\sqrt{y}}{\sqrt{y} - \sqrt{6}}$$

At this point, this expression can be simplified as follows:

$$\frac{\sqrt{y}}{\sqrt{y} - \sqrt{6}} = \frac{\sqrt{y}}{\sqrt{y} - \sqrt{6}} \cdot \frac{\sqrt{y} + \sqrt{6}}{\sqrt{y} + \sqrt{6}}$$

$$= \frac{y + \sqrt{6y}}{y - 6}$$

You were asked to rationalize the numerator of the following expression (that is, rewrite the expression as an equivalent expression that has no radicals in the numerator).

$$\frac{7 + \sqrt{x}}{10}$$

At this point, this expression can be simplified as follows:

$$\frac{7 + \sqrt{x}}{10} = \frac{7 + \sqrt{x}}{10} \cdot \frac{7 - \sqrt{x}}{7 - \sqrt{x}}$$

$$= \frac{49 - x}{70 - 10\sqrt{x}}$$

You were asked to rationalize the numerator of the following expression (that is, rewrite the expression as an equivalent expression that has no radicals in the numerator).

$$\frac{7 + \sqrt{x}}{10}$$

At this point, this expression can be simplified as follows:

$$\frac{7 + \sqrt{x}}{10} = \frac{7 + \sqrt{x}}{10} \cdot \frac{7 - \sqrt{x}}{7 - \sqrt{x}}$$

$$= \frac{49 - x}{70 - 10\sqrt{x}}$$

1.2d

Rational Number Exponents

Meaning of $a^{\frac{1}{n}}$: If n is a natural number and if $\sqrt[n]{a}$ is a real number, then $a^{\frac{1}{n}} = \sqrt[n]{a}$.

Meaning of $a^{\frac{m}{n}}$: If m and n are natural numbers with $n \neq 0$, if m and n have no common factors greater than 1, and if $\sqrt[n]{a}$ is a real number then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a}\,)^m$. Either $\sqrt[n]{a^m}$ or $(\sqrt[n]{a}\,)^m$ can be used to evaluate $a^{\frac{m}{n}}$, as they are equal. $a^{\frac{-m}{n}}$ is defined to be $\dfrac{1}{a^{\frac{m}{n}}}$.

You were asked to simplify the following expression.

$$256^{\frac{-3}{4}}$$

Before simplifying this radical expression, it may be helpful to review the Rational Number Exponents and the Properties of Radicals.

This expression can be simplified as follows:

$$\begin{aligned} 256^{\frac{-3}{4}} &= \frac{1}{256^{\frac{3}{4}}} \\ &= \frac{1}{(\sqrt[4]{256}\,)^3} \qquad \sqrt[n]{a} = a^{\frac{1}{n}} \\ &= \frac{1}{(4)^3} \\ &= \frac{1}{64} \end{aligned}$$

Answer: $\dfrac{1}{64}$

You were asked to simplify the following radical expression.

$$\sqrt[4]{3}\ \ \sqrt[16]{3}$$

Before simplifying this radical expression, it may be helpful to review the Rational Number Exponents and the Properties of Radicals.
At this point, this expression can be simplified as follows:

$$\sqrt[4]{3}\ \ \sqrt[16]{3} = 3^{\frac{1}{4}} \cdot 3^{\frac{1}{16}} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m}\,.$$

$$= 3^{\frac{1}{4}+\frac{1}{16}}$$

$$= 3^{\frac{4}{16}+\frac{1}{16}}$$

$$= 3^{\frac{5}{16}}$$

$$= \sqrt[16]{3^5}$$

$$= \sqrt[16]{243}$$

You were asked to simplify the following expression.

$$(5y^2+4)^{\frac{2}{3}}\ (5y^2+4)^{\frac{-1}{3}}$$

Before simplifying this radical expression, it may be helpful to review the Rational Number Exponents

This expression can be simplified as follows:

$$(5y^2+4)^{\frac{2}{3}}\ (5y^2+4)^{\frac{-1}{3}} = (5y^2+4)^{\frac{2}{3}+\frac{-1}{3}}$$

$$= (5y^2+4)^{\frac{1}{3}}$$

You were asked to simplify the following radical expression.

$$\sqrt[3]{x^{10}}\ \sqrt[9]{x^6}$$

Before simplifying this radical expression, it may be helpful to review the Properties of Radicals.

At this point, this expression can be simplified as follows:

$$\begin{aligned}
\sqrt[3]{x^{10}}\ \sqrt[9]{x^6} &= \sqrt[3]{x \cdot x^9} \cdot \sqrt[9]{x^6} \\
&= \sqrt[3]{x} \cdot x^3 \cdot x^{\frac{6}{9}} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} \\
&= \sqrt[3]{x} \cdot x^3 \cdot x^{\frac{2}{3}} \\
&= \sqrt[3]{x} \cdot x^3 \cdot \sqrt[3]{x^2} \\
&= x^3\ \sqrt[3]{x^3} \\
&= x^4
\end{aligned}$$

1.4

You were asked to simplify the following square root expression.

$$\left(\sqrt{-50}\right)\left(\sqrt{-2}\right)$$

This complex expression can be simplified in the following manner.

$$\begin{aligned}
\left(\sqrt{-50}\right)\left(\sqrt{-2}\right) &= \left(i\sqrt{50}\right)\left(i\sqrt{2}\right) \\
&= i^2\sqrt{50 \cdot 2} \\
&= (-1)\sqrt{100} \\
&= (-1)(10) \\
&= -10
\end{aligned}$$

You were asked to simplify the following complex expression by adding, subtracting, or multiplying, as indicated.

$$(4-8i)(4+8i)$$

The complex expression can be simplified in the following manner.

$$\begin{aligned}(4-8i)(4+8i) &= 16+32i-32i-64i^2\\ &= 16+(32-32)i-64(-1)\\ &= 16+0i+64\\ &= 80\end{aligned}$$

You were asked to simplify the following expression.

$$\frac{10}{1+3i}$$

The complex expression can be simplified in the following manner.

$$\begin{aligned}\frac{10}{1+3i} &= \left(\frac{10}{1+3i}\right)\left(\frac{1-3i}{1-3i}\right)\\ &= \frac{10(1-3i)}{(1+3i)(1-3i)}\\ &= \frac{10(1-3i)}{1+3i-3i-9i^2}\\ &= \frac{10(1-3i)}{1-9(-1)}\\ &= \frac{10(1-3i)}{10} = 1-3i\end{aligned}$$

1 - 3i is the complex conjugate of the denominator 1 + 3i

1.5a

You were asked to solve the following absolute value equation. If needed, write your answer as a fraction reduced to lowest terms.

$$| 4x - 4 | = | 3x + 2 |$$

This absolute value equation can be solved as follows:

$$| 4x - 4 | = | 3x + 2 |$$

$$4x - 4 = \pm (3x + 2) \qquad -(4x - 4) = \pm (3x + 2)$$

$$4x - 4 = 3x + 2 \quad \text{or} \quad -(4x - 4) = 3x + 2$$

$$4x - 3x = 2 + 4 \qquad -4x - 3x = 2 - 4$$

$$-7x = -2$$

$$x = 6 \qquad x = \frac{2}{7}$$

You were asked to solve the following linear equation.

$$0.9z + 3.3 = 2z$$

This linear equation can be solved as follows:

$$0.9z + 3.3 = 2z$$

$$0.9z - 2z = -3.3 \qquad \text{Add } -2z \text{ and } -3.3 \text{ to both sides.}$$

$$-1.1z = -3.3$$

$$z = 3 \qquad \text{Divide both sides by } -1.1.$$

You were asked to solve the following absolute value equation. If needed, write your answer as a fraction reduced to lowest terms.

$$|\ 5x + 4\ | + 3 = 1$$

This absolute value equation can be solved as follows:

$$|\ 5x + 4\ | + 3 = 1$$

$$\begin{aligned} 5x + 4 + 3 &= 1 \\ 5x + 4 &= -2 \\ 5x &= -6 \\ x &= \frac{-6}{5} \end{aligned} \qquad \text{or} \qquad \begin{aligned} -(\ 5x + 4\ ) + 3 &= 1 \\ -(\ 5x + 4\ ) &= -2 \\ -\ 5x &= 2 \\ x &= \frac{-2}{5} \end{aligned}$$

Since neither of these solutions solve the original equation, this absolute value equation has no solution.

Correct Answer: No Solution $(\ \varnothing\ )$

1.5b

You were asked to solve the following application problem.

Two trucks leave a warehouse at the same time. One travels due north at an average speed of 62 miles per hour, and the other travels due south at an average speed of 64 miles per hour. After how many hours will the two trucks be 819 miles apart?

The first thing that must be done is to assign a variable to the unknown quantity in this word problem.

So, let x = the number of hours that it will take for the two trucks to be 819 miles apart.

Now that the variable has been established, the information contained in this problem can be written as the following equation.

$$62x + 64x = 819$$

This linear equation can be solved as follows:

$$\begin{aligned} 62x + 64x &= 819 \\ 126x &= 819 \\ x &= \frac{819}{126} \\ x &= 6.5 \end{aligned}$$

Therefore, the two trucks will be 819 miles apart after 6.5 hours.

1.6

You were asked to solve the following compound inequality and describe the solution set using interval notation.

$$-18 < 2z - 6 \le 14$$

This compound inequality can be solved as follows:

$$-18 < 2z - 6 \le 14$$

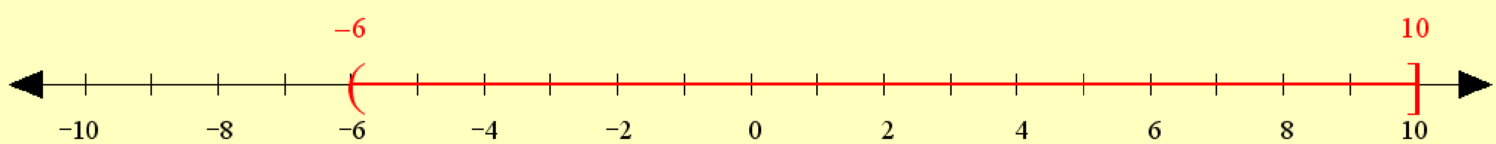
$$-18 + 6 < 2z \le 14 + 6$$

$$-12 < 2z \le 20$$

$$\frac{-12}{2} < z \le \frac{20}{2}$$

$$-6 < z \le 10$$

Placing this solution in interval notation results in $(-6, 10]$.



You were asked to solve the following linear inequality and describe the solution set using interval notation.

$$7n - 35 < -20 + 4n$$

This linear inequality can be solved as follows:

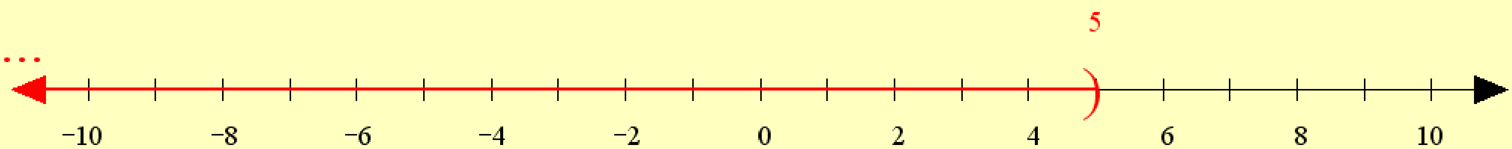
$$7n - 35 < -20 + 4n$$

$$7n - 4n < -20 + 35$$

$$3n < 15$$

$$n < 5$$

Placing this solution in interval notation results in $(-\infty, 5)$.



You were asked to solve the following compound inequality and describe the solution set using interval notation.

$$\frac{12}{8} < \frac{z+9}{4} < \frac{26}{8}$$

This compound inequality can be solved as follows:

$$\frac{12}{8} < \frac{z+9}{4} < \frac{26}{8}$$

$$8\left(\frac{12}{8}\right) < 8\left(\frac{z+9}{4}\right) < 8\left(\frac{26}{8}\right) \quad \text{Multiply each expression by the LCD.}$$

$$12 < 2(z+9) < 26$$

$$6 < (z+9) < 13$$

$$6-9 < z < 13-9$$

$$-3 < z < 4$$

Placing this solution in interval notation results in $(-3, 4)$.

1.7a

You were asked to solve the following quadratic equation by using the quadratic formula and, if needed, to submit your answer as a fraction reduced to lowest terms.

$$5z^2 + 6z - 4 = 4z$$

The first thing that needs to be done in solving a quadratic equation by using the quadratic formula is to place the quadratic equation in proper form: $az^2 + bz + c = 0$.

$$5z^2 + 2z - 4 = 0$$

This quadratic equation can be solved, using the quadratic formula, as follows:

$$5z^2 + 2z - 4 = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$z = \frac{-(2) \pm \sqrt{(2)^2 - (4)(5)(-4)}}{2(5)}$$

You were asked to solve the following quadratic equation by using the quadratic formula and, if needed, to submit your answer as a fraction reduced to lowest terms.

$$5z^2 + 6z - 4 = 4z$$

$$z = \frac{-2 \pm \sqrt{4 + 80}}{10}$$

$$z = \frac{-2 \pm \sqrt{84}}{10}$$

$$z = \frac{-2 \pm 2\sqrt{21}}{10}$$

$$z = \frac{-1 + \sqrt{21}}{5}, \quad \frac{-1 - \sqrt{21}}{5}$$

Answer: $\frac{-1 + \sqrt{21}}{5}$ or $\frac{-1 - \sqrt{21}}{5}$

You were asked to solve the following quadratic equation by factoring and, if needed, to submit your answer as a fraction reduced to lowest terms.

$$y^2 + 10y + 16 = 0$$

The first thing that needs to be done in solving a quadratic equation by factoring is to place the quadratic equation in proper form: $ax^2 + bx + c = 0$.

Next, since $a = 1$, we need to find two factors of 16 such that the sum of the two factors is 10.
So, this quadratic equation can be solved by factoring as follows:

$$y^2 + 10y + 16 = 0 \qquad \text{8 and 2 are the correct factors of 16,}$$
$$(y + 8)(y + 2) = 0 \qquad \text{since } 8 \cdot 2 = 16 \text{ and } 8 + 2 = 10.$$
$$y + 8 = 0 \quad \text{or} \quad y + 2 = 0$$
$$y = -8 \quad \text{or} \quad y = -2$$

Answer: -8 or -2

1.7b

You were asked to solve the following quadratic-like equation and, if needed, to submit your answer as a fraction reduced to lowest terms.

$$7y^{\frac{12}{5}} - 13y^{\frac{7}{5}} + 6y^{\frac{2}{5}} = 0$$

The first step that needs to be taken is to recognize that each term in this equation has a common factor, $y^{\frac{2}{5}}$.

This equation can be factored as follows:

$$7y^{\frac{12}{5}} - 13y^{\frac{7}{5}} + 6y^{\frac{2}{5}} = 0$$

$$y^{\frac{2}{5}} \left(7y^2 - 13y + 6 \right) = 0$$

$$y^{\frac{2}{5}} \left(7y - 6 \right)\left(y - 1 \right) = 0$$

Now that this equation has been factored, the three resulting equations can be solved in the following manner.

$$y^{\frac{2}{5}} = 0 \qquad \left(7y - 6 \right) = 0 \qquad \left(y - 1 \right) = 0$$

$$y = 0 \qquad 7y = 6 \qquad y = 1$$

$$y = \frac{6}{7}$$

You were asked to solve the following quadratic-like equation and, if needed, to submit your answer as a fraction reduced to lowest terms.

$$7z^{\frac{11}{5}} - 37z^{\frac{6}{5}} + 10z^{\frac{1}{5}} = 0$$

The first step that needs to be taken is to recognize that each term in this equation has a common factor, $z^{\frac{1}{5}}$.

This equation can be factored as follows:

$$7z^{\frac{11}{5}} - 37z^{\frac{6}{5}} + 10z^{\frac{1}{5}} = 0$$

$$z^{\frac{1}{5}} \left(7z^2 - 37z + 10 \right) = 0$$

$$z^{\frac{1}{5}} \left(7z - 2 \right)\left(z - 5 \right) = 0$$

Now that this equation has been factored, the three resulting equations can be solved in the following manner.

$$z^{\frac{1}{5}} = 0 \qquad (7z - 2) = 0 \qquad (z - 5) = 0$$

$$z = 0 \qquad 7z = 2 \qquad z = 5$$

$$z = \frac{2}{7}$$

1.8a NOT ON THE TEST

You were asked to simplify the following rational expression.

$$\frac{z^2 + 2z - 8}{z^2 - 8z + 12}$$

The first step that needs to be taken is to completely factor both the numerator and the denominator of this rational expression.

$$\frac{z^2 + 2z - 8}{z^2 - 8z + 12} = \frac{(z + 4)(z - 2)}{(z - 2)(z - 6)}$$

Once this rational expression has been completely factored, as above, it should be clear that there is a common factor of $(z - 2)$ in both the numerator and the denominator.

Canceling this common factor results in the following rational expression.

$$\frac{z + 4}{z - 6}$$

Answer: $\frac{z + 4}{z - 6}$

You were asked to solve the following work-rate problem and, if needed, submit your answer as a fraction reduced to lowest terms.

Officials begin to release water from a full man-made lake at a rate that would empty the lake in 10 weeks, but a river that can fill the lake in 20 weeks is replenishing the lake at the same time. How long does it take to empty the lake?

The rate of work for the apparatus used to remove the water from the lake is $\frac{1}{10}$, while the rate of work for the river, which is replenishing the lake, is $-\frac{1}{20}$. The rate of work for the river is negative, since it is performing the opposite function of the apparatus used to remove the water from the lake, whose rate of work is positive.

If we let x denote the time needed to remove the water from the lake, the sum of the two individual rates must equal $\frac{1}{x}$.

So, we need to solve the equation $\frac{1}{10} - \frac{1}{20} = \frac{1}{x}$.

In this case, the LCD is $20x$.

At this point, this equation can be solved as follows:

$$20x \cdot \frac{1}{10} - 20x \cdot \frac{1}{20} = 20x \cdot \frac{1}{x}$$

$$2x - x = 20$$

$$x = 20$$

Answer: 20 weeks

1.8b NOT ON THE TEST

You were asked to solve the following radical equation and, if needed, submit your answer as a fraction reduced to lowest terms.

$$\sqrt{3y + 51} + 9 = y + 8$$

This radical equation can be solved in the following manner.

$$\sqrt{3y + 51} + 9 = y + 8$$

$$\sqrt{3y + 51} = y - 1$$

$$\left(\sqrt{3y + 51} \right)^2 = (y - 1)^2$$

$$3y + 51 = y^2 - 2y + 1$$

$$0 = y^2 - 5y - 50$$

$$0 = (y - 10) (y + 5)$$

$$y = 10, -5$$

At this point, both of these possible solutions must be checked in the original equation.

$$\sqrt{3y+51}+9 = y+8$$
$$\sqrt{3y+51} = y-1$$
$$\sqrt{3(10)+51} = (10)-1$$
$$\sqrt{81} = 9$$
$$9 = 9$$

This is True, therefore $y = 10$

$$\sqrt{3y+51}+9 = y+8$$
$$\sqrt{3y+51} = y-1$$
$$\sqrt{3(-5)+51} = (-5)-1$$
$$\sqrt{36} = -6$$
$$6 \neq -6$$

This is False, therefore $y \neq -5$

Answer: 10