

Standard Form of a Circle

The **standard form** of the equation for a circle of radius r with center (h, k) is

$$(x - h)^2 + (y - k)^2 = r^2.$$

THEOREM

The standard form of an equation of a circle of radius r with center at the origin $(0, 0)$ is

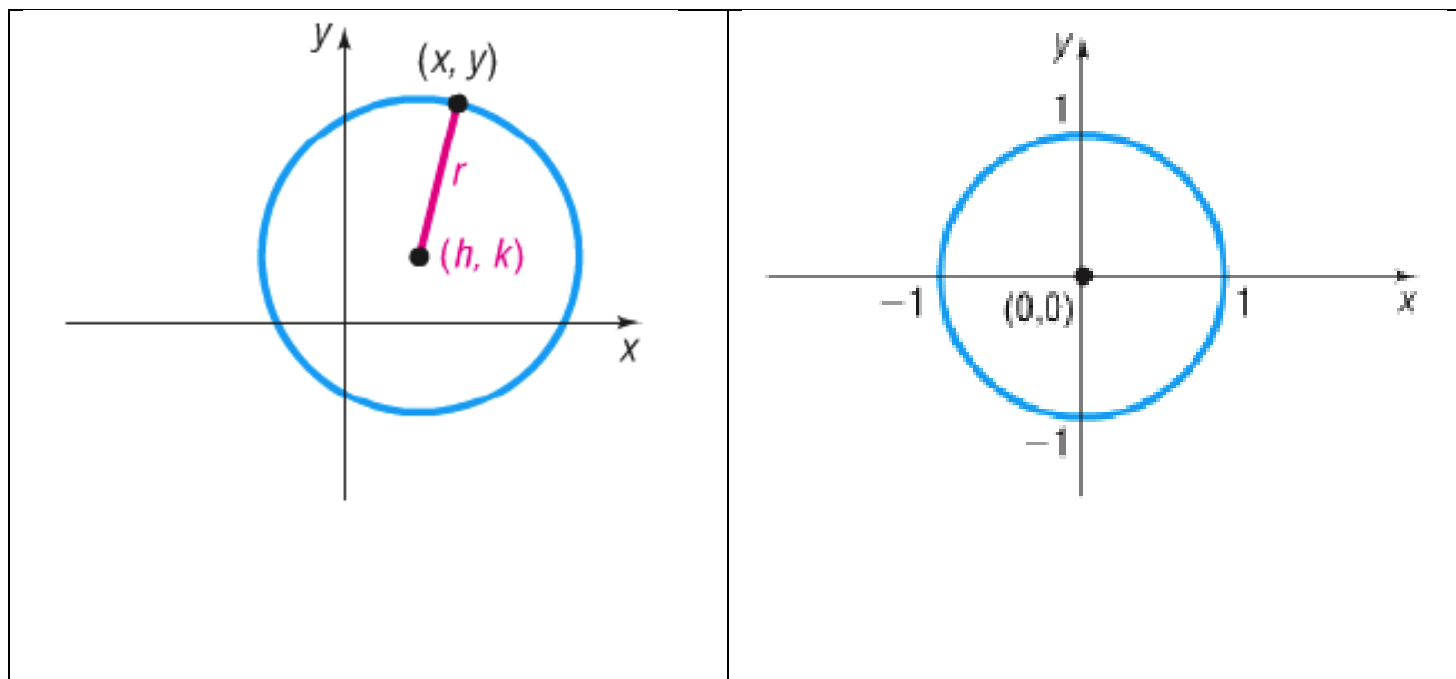
$$x^2 + y^2 = r^2$$

DEFINITION

If the radius $r = 1$, the circle whose center is at the origin is called the **unit circle** and has the equation

$$x^2 + y^2 = 1$$

See Figure 50. Notice that the graph of the unit circle is symmetric with respect to the x -axis, the y -axis, and the origin.



Writing the Standard Form of the Equation of a Circle

Write the standard form of the equation of the circle with radius 5 and center $(-3, 6)$.

Using equation (1) and substituting the values $r = 5$, $h = -3$, and $k = 6$, we have

$$\begin{aligned}(x - h)^2 + (y - k)^2 &= r^2 \\(x + 3)^2 + (y - 6)^2 &= 25\end{aligned}$$

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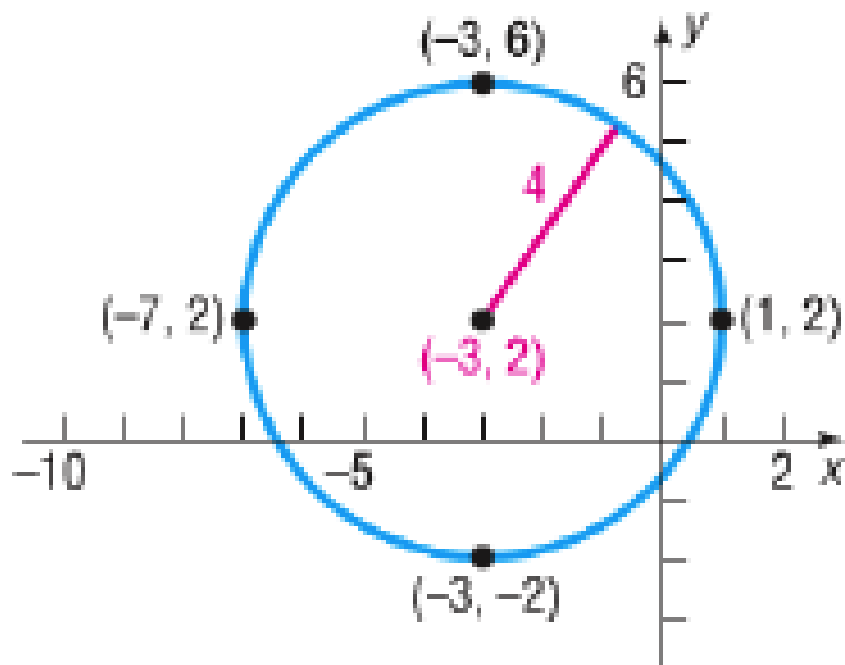
Graph the equation: $(x + 3)^2 + (y - 2)^2 = 16$

Since the equation is in the form of equation (1), its graph is a circle. To graph the equation, compare the given equation to the standard form of the equation of a circle. The comparison yields information about the circle.

$$\begin{aligned}(x + 3)^2 + (y - 2)^2 &= 16 \\(x - (-3))^2 + (y - 2)^2 &= 4^2 \\(x - h)^2 + (y - k)^2 &= r^2\end{aligned}$$

We see that $h = -3$, $k = 2$, and $r = 4$. The circle has center $(-3, 2)$ and a radius of 4 units. To graph this circle, first plot the center $(-3, 2)$. Since the radius is 4, we can locate four points on the circle by plotting points 4 units to the left, to the right, up, and down from the center. These four points can then be used as guides to obtain the graph. See Figure 51.

Figure 51



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Work with the General Form of the Equation of a Circle

If we eliminate the parentheses from the standard form of the equation of the circle given in Example 2, we get

$$\begin{aligned}(x + 3)^2 + (y - 2)^2 &= 16 \\ x^2 + 6x + 9 + y^2 - 4y + 4 &= 16\end{aligned}$$

which, upon simplifying, is equivalent to

$$x^2 + y^2 + 6x - 4y - 3 = 0 \quad (2)$$

It can be shown that any equation of the form

$$x^2 + y^2 + ax + by + c = 0$$

has a graph that is a circle, or a point, or has no graph at all. For example, the graph of the equation $x^2 + y^2 = 0$ is the single point $(0, 0)$. The equation $x^2 + y^2 + 5 = 0$, or $x^2 + y^2 = -5$, has no graph, because sums of squares of real numbers are never negative.

When its graph is a circle, the equation

$$x^2 + y^2 + ax + by + c = 0$$

is referred to as the **general form of the equation of a circle**.

Graphing a Circle Whose Equation Is in General Form

Graph the equation $x^2 + y^2 + 4x - 6y + 12 = 0$

Group the terms involving x , group the terms involving y , and put the constant on the right side of the equation. The result is

$$(x^2 + 4x) + (y^2 - 6y) = -12$$

Next, complete the square of each expression in parentheses. Remember that any number added on the left side of the equation must also be added on the right.

$$(x^2 + 4x + 4) + (y^2 - 6y + 9) = -12 + 4 + 9$$

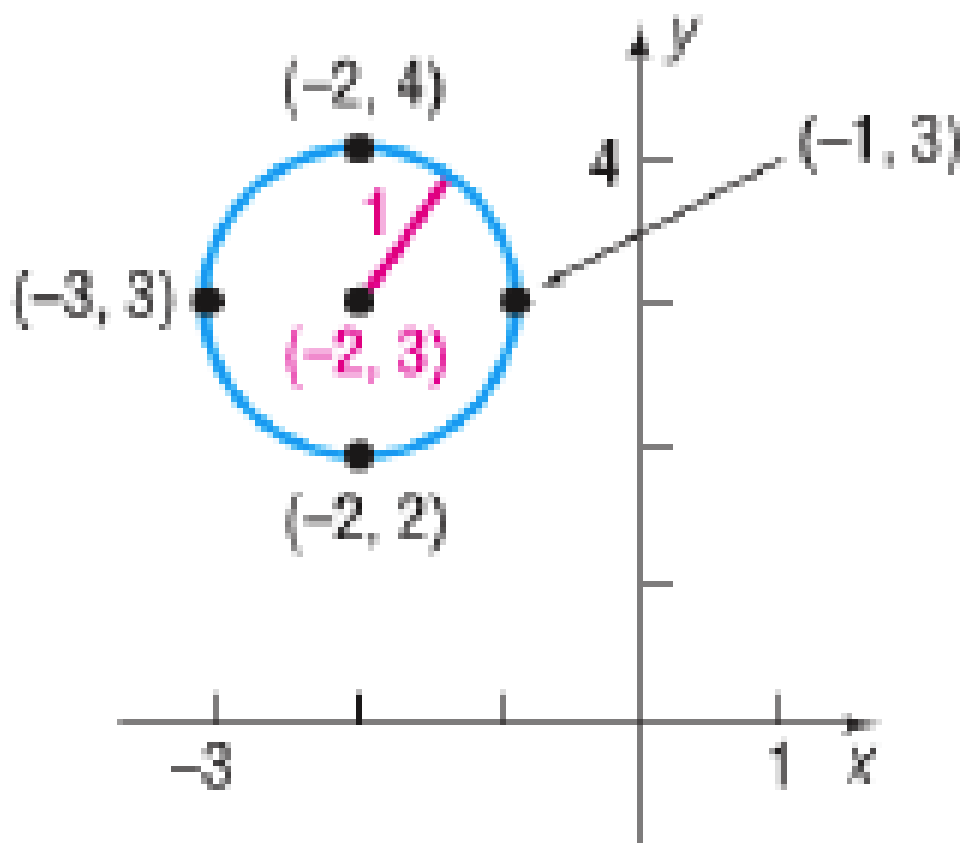
$$\underbrace{\hspace{1.5cm}}_{\left(\frac{4}{2}\right)^2 = 4} \quad \underbrace{\hspace{1.5cm}}_{\left(\frac{-6}{2}\right)^2 = 9}$$

$$(x + 2)^2 + (y - 3)^2 = 1 \quad \text{Factor.}$$

This equation is the standard form of the equation of a circle with radius 1 and center $(-2, 3)$.

To graph the equation use the center $(-2, 3)$ and the radius 1. See Figure 52. |

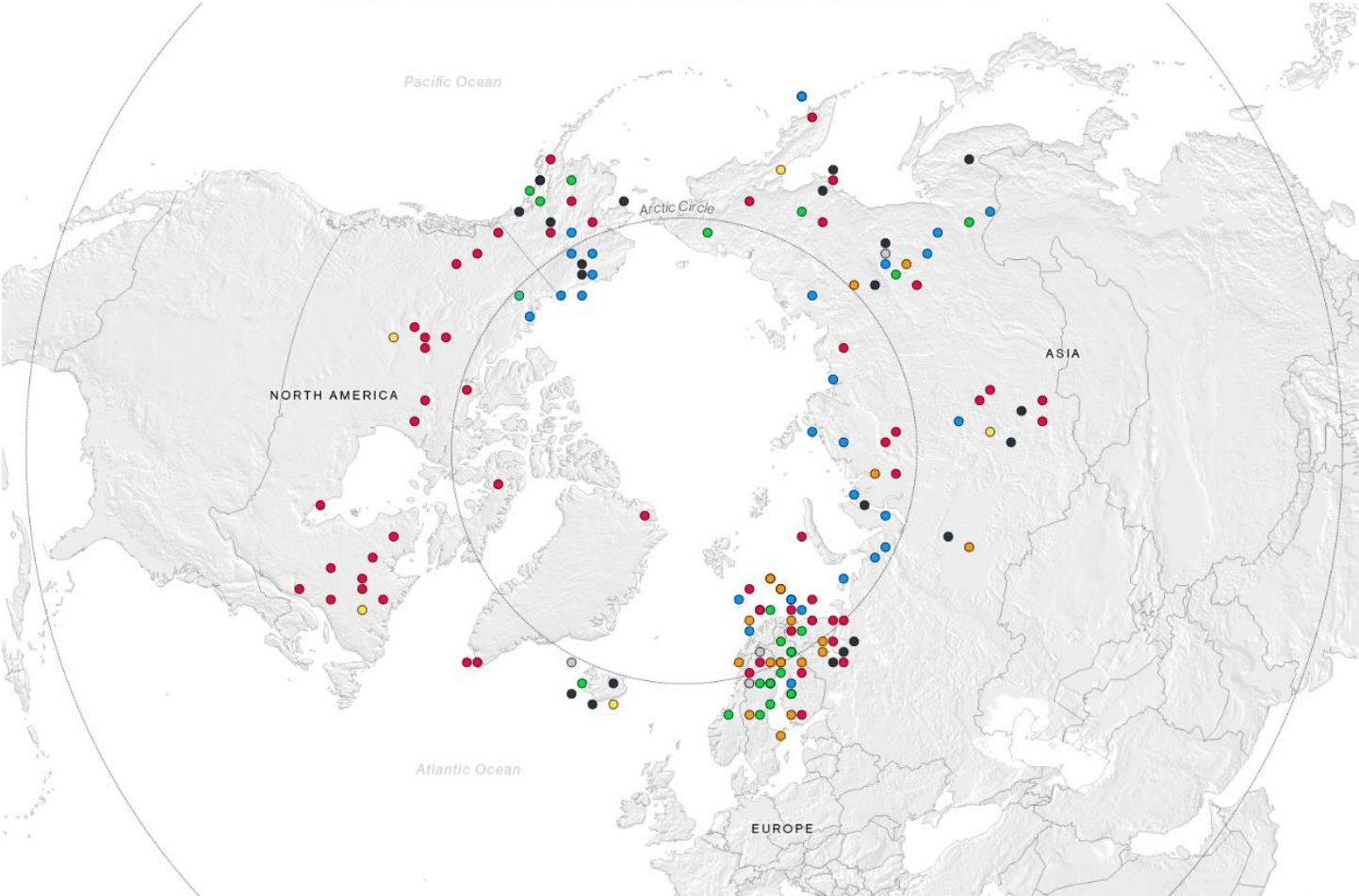
Figure 52



REAL LIFE EXAMPLE OF CIRCLES

Development in the Arctic
From roads to new mines, these are some of the largest projects on each country's wish list, according to Guggenheim Partners:

- Transportation
- Fossil Fuel Energy
- Renewable/Nuclear Energy
- Mining
- Power
- Economic
- Civic



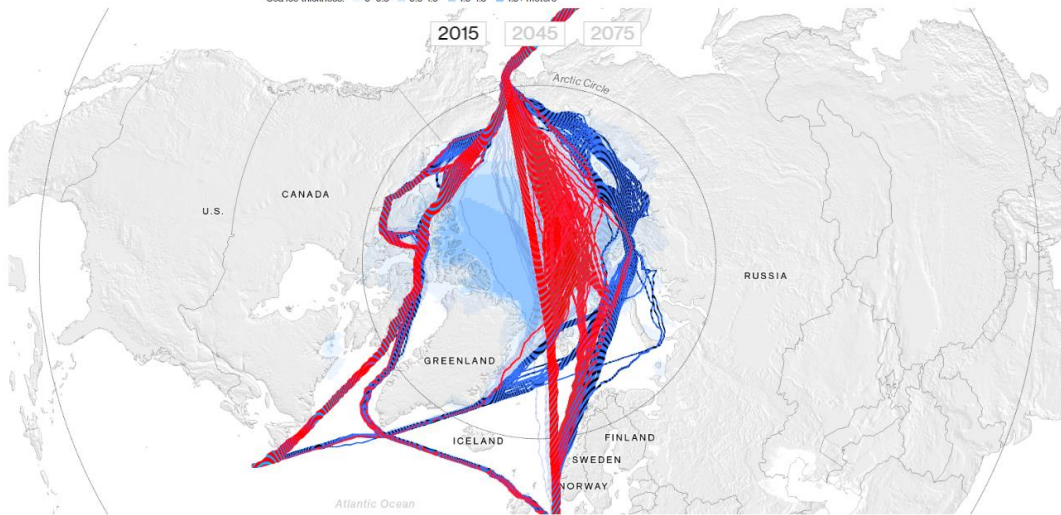
Potential Projects, and Their Estimated Cost, by Country



Source: Guggenheim Partners

As the Ice Melts, Arctic Shipping Routes Expand

■ Overwater shipping routes ■ Icebreaker shipping routes
Sea ice thickness: 0-0.5 0.5-10 10-15 15+ meters



Source: Møller, N., K. Haines, and E. Hawkins, *Sea Ice Decline and 21st Century Trans-Arctic Shipping Routes*

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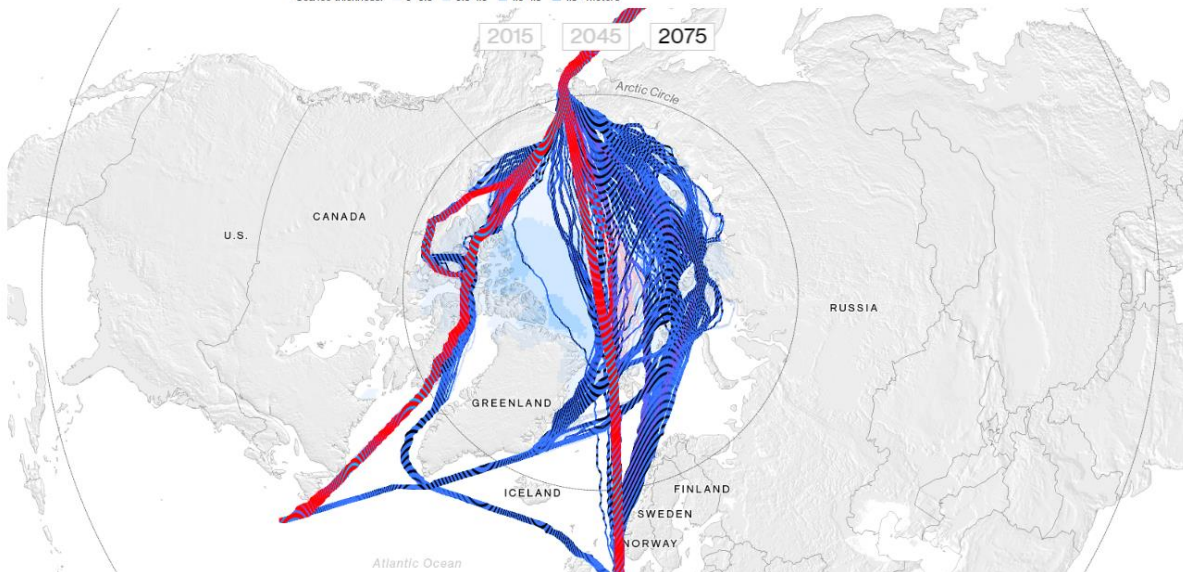
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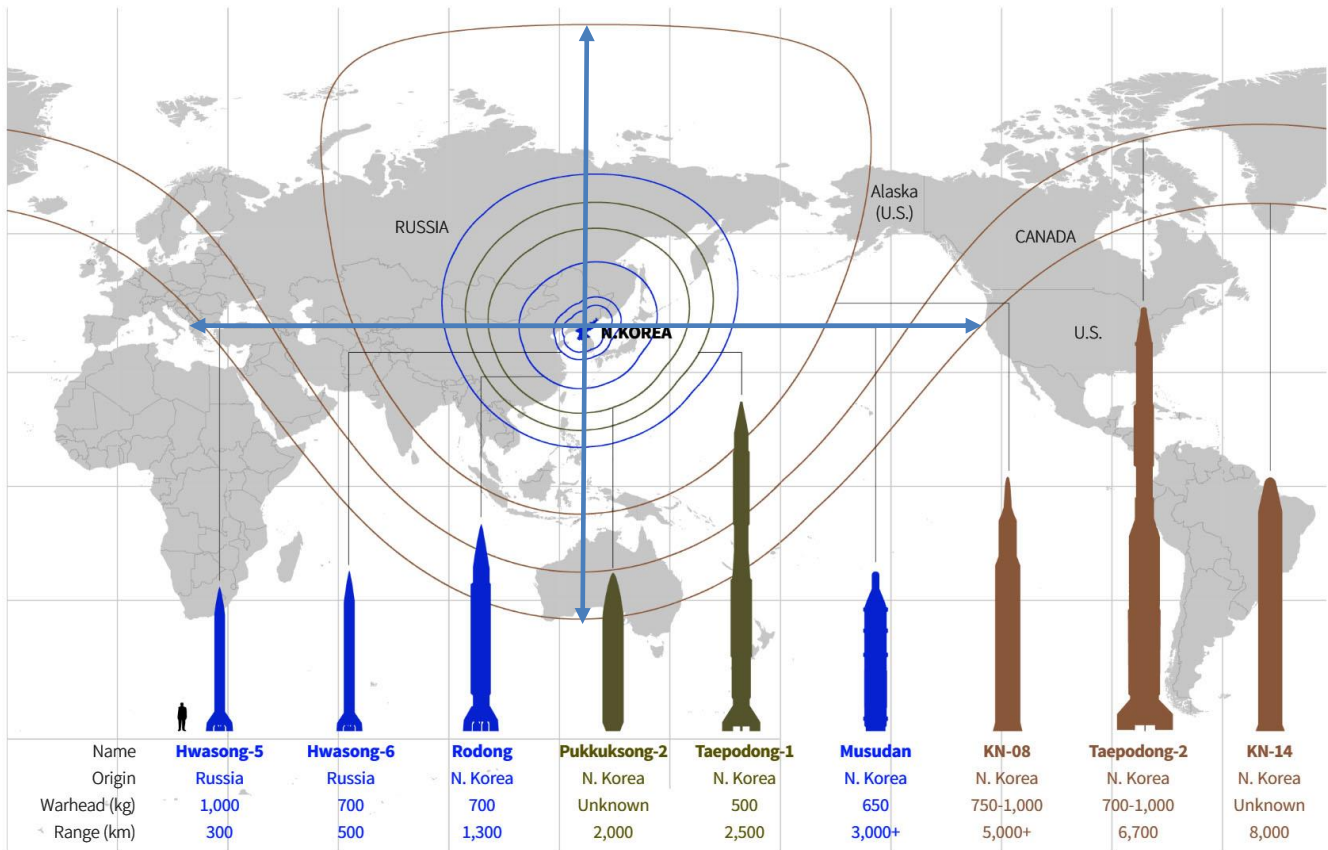
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Operational Flight tested In development or untested



1
 The distance capability of the KN-14 missile is 8 thousand km from north korea which is the center.
 Give me the circle of this analysis on mathematica and give me the standard form of the equation

A CITIZEN'S GUIDE TO SURVIVING "THE BUTTON"

Here's what would happen if the missile North Korea tested in 2017 detonated above Lower Manhattan

Detonation altitude:
5,446 feet

Manhattan
The Bronx
Queens
Jersey City
Brooklyn

- **FIREBALL | RADIUS: .28 MILES**
This would be the size of the fireball—the fallout would depend on whether it detonated on the ground, which would make fatalities much worse. Almost everyone in this area would be killed immediately.
- **RADIATION | RADIUS: .62 MILES**
Things get grim just outside of the fireball's impact, where there's between a 50 and 90 percent chance of mortality just from acute effects. Death could take hours, or even weeks.
- **AIR BLAST | RADIUS: 2.3 MILES**
Even two miles out, you're still in danger of flying debris. If caught outside, lie down, take cover, and don't look at the fireball. (It can blind you.)
- **THERMAL RADIATION | RADIUS: 3.3 MILES**
Your biggest worry at this distance would be third-degree burns, which could cause scarring at best, and amputation in the worst cases.

EST. TOTAL DEATHS: 728,220
EST. TOTAL INJURIES: 1.56 million

● What do you do when you get that push notification on your phone that a missile strike is imminent? Run inside and get to the lowest or most interior floor you can. The larger the value listed on the chart below, the safer you'll be from the blast's effects (think of it as SPF, but for nukes).

One-story wood frame

2-3
10

Two-story brick veneer

3
20

Three-story brick, concrete

7
50

Five-story brick, concrete

10
20
100
200

Ten-story apartment/office building

10
20
30
30
30
30
20
10
200

Your car is not safe.

Sources: NUKEMAP; Ready.gov; and the Lawrence Livermore National Laboratory.

Exercises

Find the standard form of the equation for the circle with radius 3 and center $(-2, 7)$.

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Sketch the graph of the circle defined by

$$(x - 2)^2 + (y + 3)^2 = 4.$$

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Find the standard form of the equation for the circle with radius 3 and center $(3, 1)$

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Consider the equation $x^2 + y^2 - 8y + 7 = 0$, Find the center (h, k) , and radius, r of this circle and graph the circle.

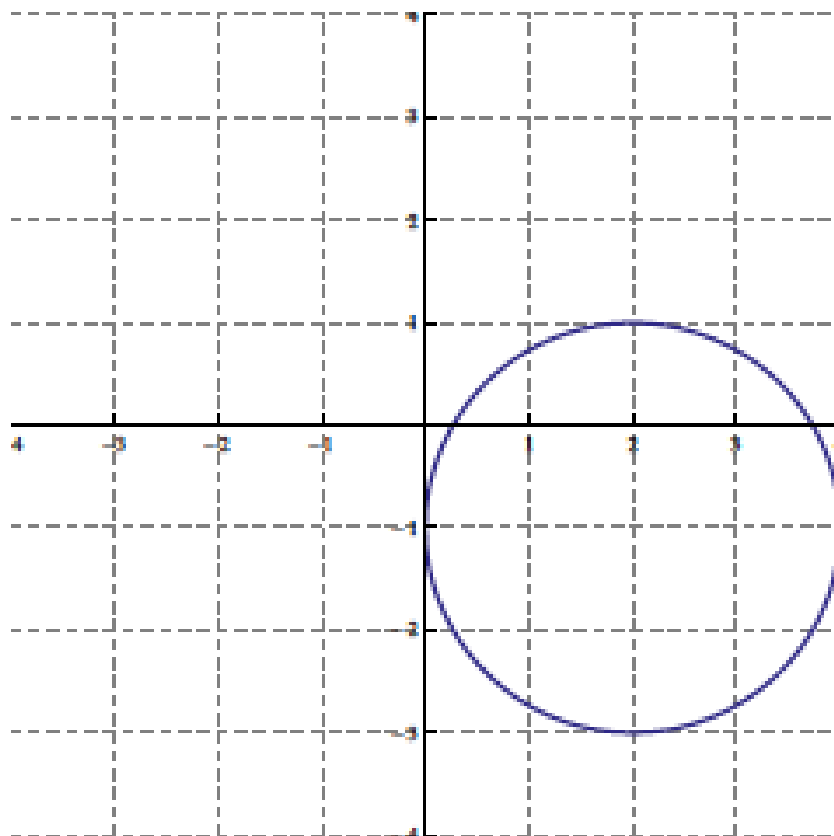
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Consider the equation $x^2 + y^2 - 14x + 10y + 38 = 0$, Find the center (h, k) , and radius, r of this circle and graph the circle.

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Consider the circle pictured below.

- Find the center (h, k) , and radius, r of this circle.
- Write the equation of the circle in standard form.
- Find the intercepts.



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A circle has the equation $7x^2 + 7y^2 - 42x - 28y - 84 = 0$. Graph the circle using the center (h,k) and radius r . Find the intercepts, if any, of the graph.

Use the graphing tool to graph the circle.



At what points do the x-intercepts occur? Select the correct choice below and fill in any answer boxes within your choice.

A. The x-intercept(s) is/are .
(Type an ordered pair. Use a comma to separate answers as needed. Type exact answers for each coordinate, using radicals as needed.)

B. There is no x-intercept.

At what points do the y-intercepts occur? Select the correct choice below and fill in any answer boxes within your choice.

A. The y-intercept(s) is/are .
(Type an ordered pair. Use a comma to separate answers as needed. Type exact answers for each coordinate, using radicals as needed.)

B. There is no y-intercept.

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