reciprocal identities

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x}$$

quotient identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

pythagorean identities

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

sec (x) cos (x)

$$Answer: \frac{1}{\cos(x)} * \cos(x) = 1$$

$$\frac{1}{\sec^2(Á) - 1}$$

$$\frac{1}{\tan^2 \beta} \text{ or}$$

$$\text{Answer} := \cot^2(x)$$

$$\frac{\sec(\theta)}{\csc(\theta)}$$

$$\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} = \tan\theta$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

Exercises

cos u sec u – cos ^2 u = sin ^2 u

$$\frac{\cos u}{1} \times \frac{1}{\cos u} - \frac{\cos^2 u}{1}$$

$$\frac{1 - \cos^2 u}{1}$$

sin^2 u = sin^2 u → True

Tan u (csc u – sin u) = cos u

tan u csc u – tan u sin u

$$\frac{\sin u}{\cos u} \times \frac{1}{\sin u} - \frac{\sin u}{\cos u} \times \frac{\sin u}{1}$$

$$\frac{1}{\cos u} - \frac{\sin^2 u}{\cos u}$$

$$\frac{1-\sin^2 u}{\cos u} \rightarrow \frac{\cos^2 u}{\cos u} \rightarrow$$ cos u = cosu → True

Chapter 7 (7.5) Sum and difference identities

sum and difference identities

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Sum and Difference Formulas

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

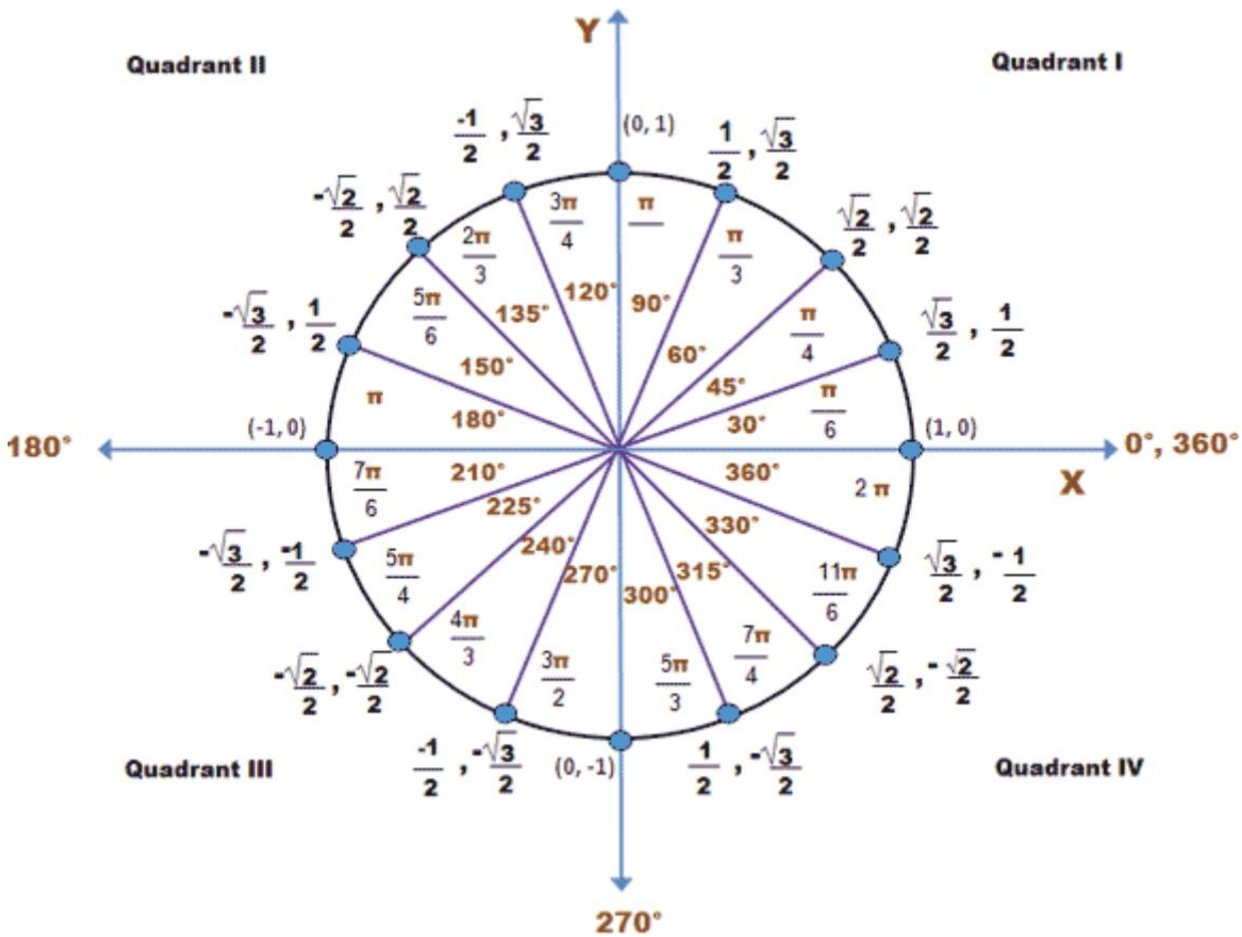
$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Sum and Difference Formulas for the Cosine Function

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \textbf{(1)}$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \qquad \textbf{(2)}$$



$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$	$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
Use a sum or difference to find the exact value of the trig function. **Cos 195** Rewrite 195 as the sum of two angles. 30,45,60,90 Angles which correspond to the circle for Cos $\cos 195^\circ = \cos(135^\circ + 60^\circ)$ $= \cos 135^\circ \cos 60^\circ - \sin 135^\circ \sin 60^\circ$ $\cos 195^\circ = \left(-\frac{\sqrt{2}}{2}\right) \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$ $= -\frac{1}{4}(\sqrt{2} + \sqrt{6})$	$\cos(\frac{\pi}{6})\cos(\frac{3\pi}{5}) + \sin(\frac{\pi}{6})\sin(\frac{3\pi}{5})$ Cos (30) cos (108) + sin (30) sin (108) Not on the graph circle because of 108 Cos (pi/6 – 3pi/5) Cos (30 - 108) COS (-78) There is no angle on the circle leave as is.

$\cos \frac{13\pi}{12}$ or	**Cos 195** **See process above**

Rewrite as the sum of two trig values

$\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2},$

$$\frac{13\pi}{12} = \frac{4\pi}{12} + \frac{9\pi}{12} = \frac{\pi}{3} + \frac{3\pi}{4}$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos\frac{13\pi}{12} = \cos\left(\frac{\pi}{3} + \frac{3\pi}{4}\right)$$

$$\cos\frac{\pi}{3}\cos\frac{3\pi}{4} - \sin\frac{\pi}{3}\sin\frac{3\pi}{4}$$

$$\cos\frac{13\pi}{12} = \frac{1}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$-\frac{1}{4}\left(\sqrt{2}+\sqrt{6}\right)$	**Exact value**

Cos (pi/12) cos (5pi/12) + sin (pi/12) sin (5pi/12)

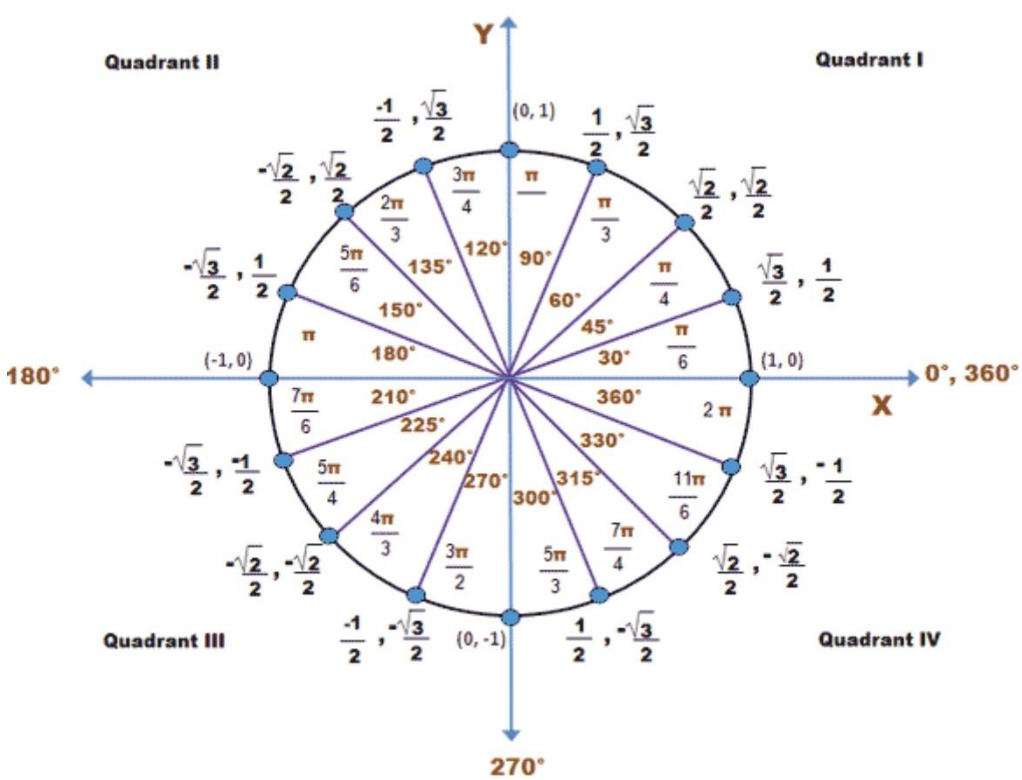
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Cos (pi/12 - 5pi/12)

Cos (-4pi /12) → cos (-60)

evaluate

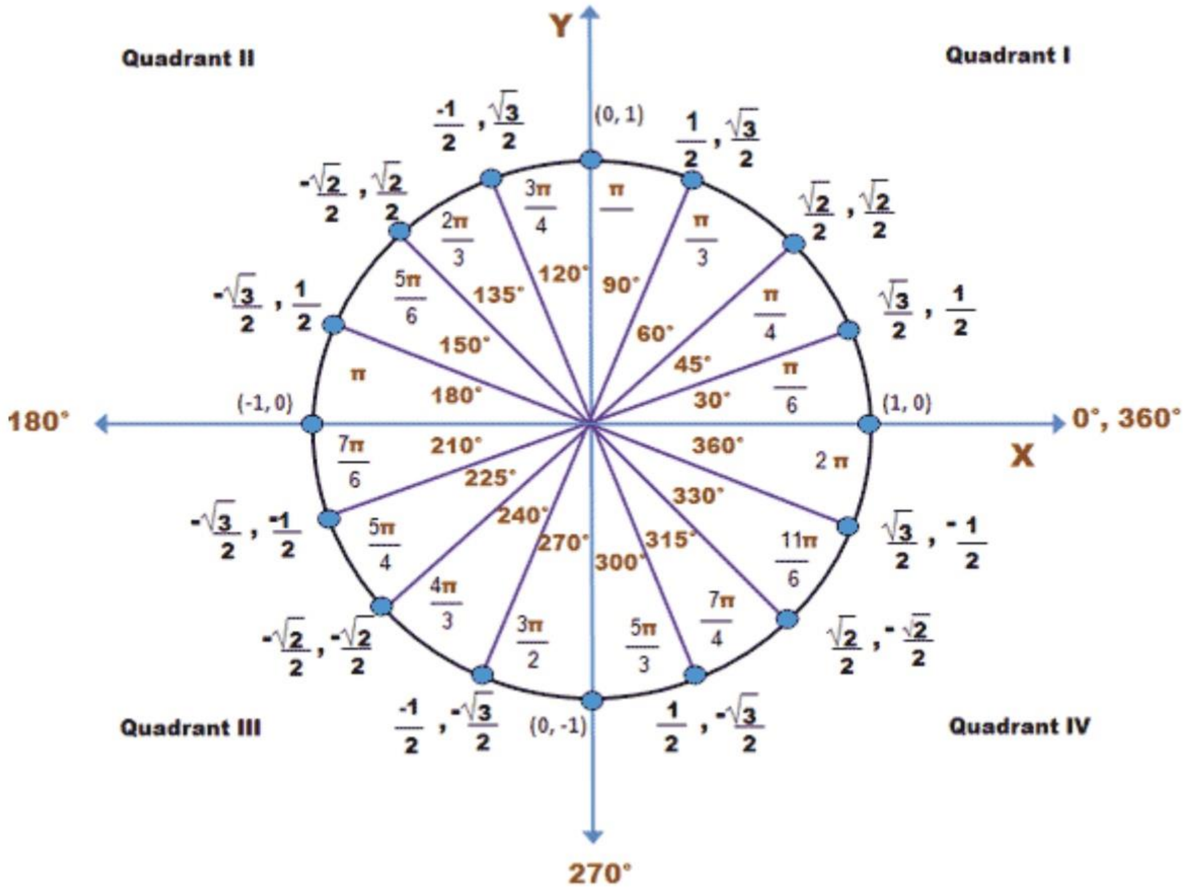
½ from graph



Sum and Difference Formulas for the Sine Function

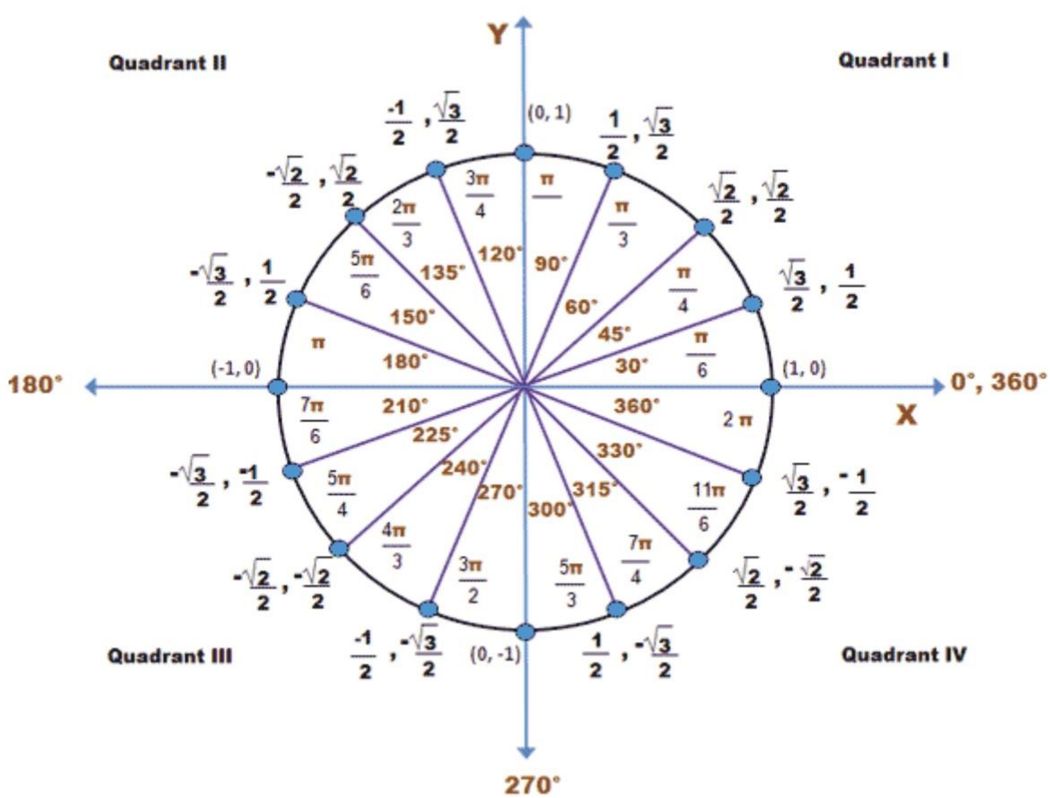
$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad (4)$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \qquad (5)$$



$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$	$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\cos(\frac{\pi}{4})\sin(\frac{\pi}{3}) + \sin(\frac{\pi}{4})\cos(\frac{\pi}{3})$ Sin(a+b) Sin (pi/3 + pi/4) Sin (4pi/12 + 3pi/12) sin (7pi/12) sin (105) not on the cirle leave as is.	$\sin\frac{\pi}{2}\cos\frac{3\pi}{4} - \cos\frac{\pi}{2}\sin\frac{3\pi}{4}$ $\sin\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)$ $\sin\left(-\frac{\pi}{4}\right)$ Evaluate $-\frac{\sqrt{2}}{2}$ VALUE FROM THE CIRCLE

sin 90° cos 20° + cos 90° sin 20°. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ Sin(a+b) Sin (90 + 20) Sin 110 NO EXACT VALUE ON THE CIRCLE! LEAVE AS IS	
$\sin(120)\cos(30) + \cos(120)\sin(30)$ $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ Sin (a+b) Sin (120 + 30) Sin (150) look at the circle graph for the answer See graph Sin = ½ at 150 degrees	



Sum and Difference Formulas for the Tangent Function

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \quad \textbf{(6)}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta} \quad \textbf{(7)}$$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$	$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$
$\frac{\tan 70 + \tan 45}{1 - \tan 70 \tan 45}$ tan (70 + 45) tan (115)	Tan 75 Rewrite 75 as the difference of two angles. 30,45,60,90 75 = 120 – 45 tan 75 = tan (120-45) tan120 - tan45 tan 75 = --------------------- 1 + tan 120 tan 45
$\frac{\tan\frac{5\pi}{14} + \tan\frac{2\pi}{14}}{1 - \tan\frac{5\pi}{14}\tan\frac{2\pi}{14}}$ tan (5pi/14 + 2pi/14) tan (7pi/14)	

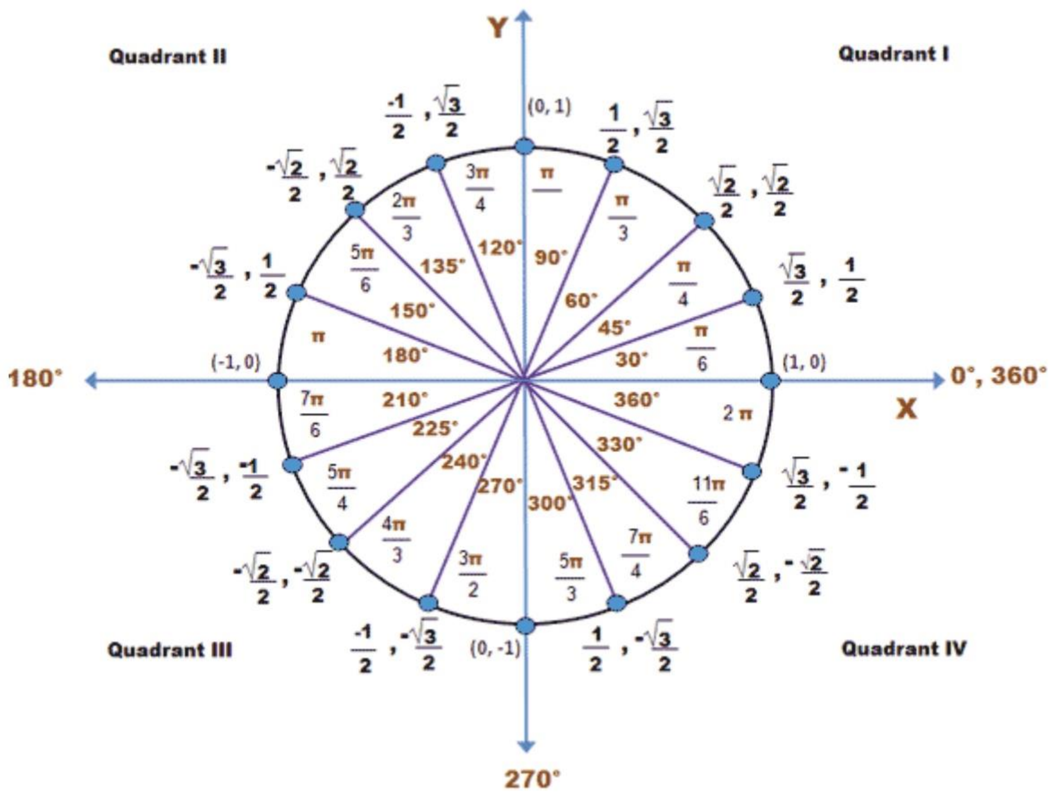
Exercises

Use the sum and difference identities to determine the exact value of the expression $\sin(-\frac{11\pi}{6})$

Sum and Difference Formulas for the Sine Function

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \quad \textbf{(4)}$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta \quad \textbf{(5)}$$



Sin (-11pi / 6) = sin (-330) Rewrite 195 as the sum of two angles. 30,45,60,90 Angles which correspond to the circle for Sin Sin (-330) Sin (-240 + -90) $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$	$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ sin(-240) cos (-90) + cos (-240) sin(-90) Sqrt 3/2 * 0 + -1/2 * -1 $0 + \frac{1}{2} \rightarrow + \frac{1}{2}$ the above was not required becaue -330 is on the circle graph and you would just have to put + ½ as your answer for sin (-330)