CHAPTER 5 SULLIVAN 9th Ed

5.1 Composite functions

Composition of Functions

Let f and g be two functions. The **composition** of f and g, denoted $f \circ g$, is the function defined by $(f \circ g)(x) = f(g(x))$. The domain of $f \circ g$ consists of all x in the domain of g for which $g(x)$ is in turn in the domain of f. The function $f \circ g$ is read "f composed with g", or "f of g."

Another way of combining functions beside multiplication, division, addition and subtraction is in the form of COMPOSITION.

COMPOSITION of one function with another means apply one function **f(x)** to the output of another function **g(x).**

In COMPOSITION functions f(x) and g(x) are not commutative [a+b = b+a or ab= ba].

The diagram in Figure 1 is a sort of schematic of the composition of two functions. The ovals represent sets, with the leftmost oval being the domain of the function g. The arrows indicate the element that x is associated with by the various functions.

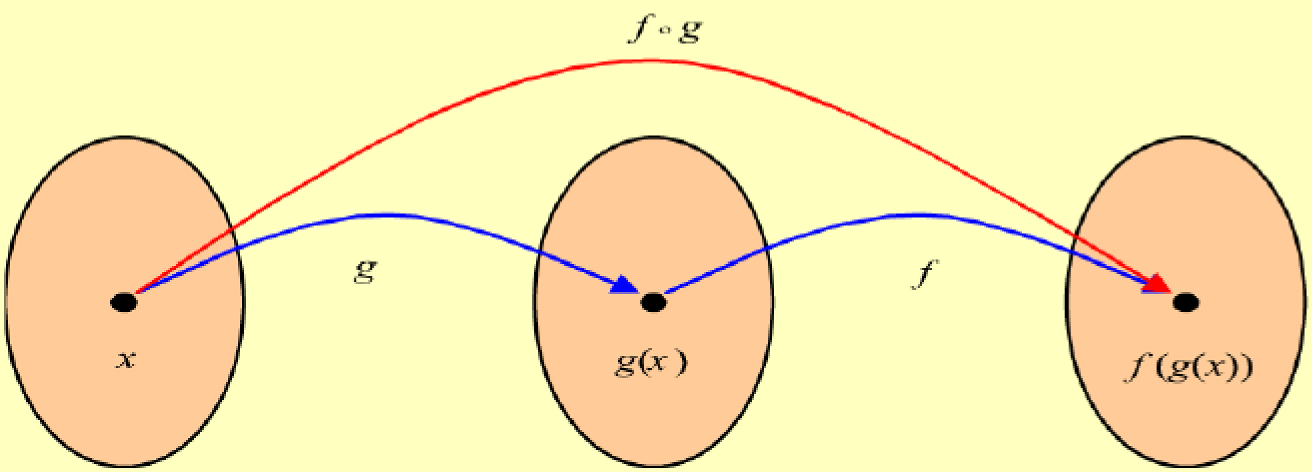

Figure 1: Composition of *f* and *g*

As with the four arithmetic ways of combining functions, we can evaluate the composition of two functions at a single point, or find a formula for the composition if we have been given formulas for the individual functions.

Given $f(x) = 2x^2 - 5$ and $g(x) = [\![\, x+2 \,]\!]$, find:	**a.** $(f \circ g)(4)$ f of g

a. $(f \circ g)(4)$

x=4 then g(x) = x +2 → **4**+2 // f(g(x)) then f(4+2)→ f(6) // f(6) = 2x^2-5 → f(6) = 2(6)^2—5 → **67**

a. $g(4) = [\![\, 4+2 \,]\!] = 6$ and so $(f \circ g)(4) = f(g(4)) = f(6) = 67.$

a. $f(x) = x^2$ and $g(x) = 5x - 8$

$f(g(x)) =$ thus f(g(x)) = f(5x-8) hence we takes this value (5x-8)^2 or (5x-8)(5x-8) = **25x^2-80x+64**

Let $f(x) = |\ x - 2\ |$ and $g(x) = \dfrac{x+1}{3}$. Find formulas and state the domains for:

a. $f \circ g$

Solution:

a. $(f \circ g)(x) = f\left(\dfrac{x+1}{3}\right)$

$$= \left|\left(\frac{x+1}{3}\right) - 2\right|$$

$$= \left|\frac{x-5}{3}\right|$$

g(x)= [(x+1)/ (3)] → f(g(x)) = f([(x+1)/ (3)])

f *g is where the value for g(x) becomes the value of "x" in f(x).

f([(x+1)/ (3)]) = |x-2| → |[(x+1)/ (3)]) -2| →

|[(x+1)/ (3)]) -2|→ |[(x+1)/ (3)]) –(2/1) (3/3) |

|[(x+1)/ (3)]) –(6/3)|

|[(x-5)/ (3)]|

Always keep the absolute value sign.

b. $g \circ f$

$f(x) = \|\ x - 2\ \|$	$g(x) = \dfrac{x+1}{3}$

b. $(g \circ f)(x) = g\left(|\ x - 2\ |\right)$

$$= \frac{|\ x-2\ | + 1}{3}$$

f(x) = |x-2| → g(f(x)) = g(|x-2|)

g *f is where the value for f(x) becomes the value of "x" in g(x).

g(|x-2|) = [(x+1)/ (3)] → **[([|x-2|]+1)/ (3)]**

Given two functions f and g, the **composite function,** denoted by $f \circ g$ (read as "f composed with g"), is defined by

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

Evaluating a Composite Function

Suppose that $f(x) = 2x^2 - 3$ and $g(x) = 4x$. Find:

(a) $(f \circ g)(1)$ (b) $(g \circ f)(1)$ (c) $(f \circ f)(-2)$ (d) $(g \circ g)(-1)$

(a) $(f \circ g)(1) = f(g(1)) = f(4) = 2 \cdot 4^2 - 3 = 29$

$g(x) = 4x$ $f(x) = 2x^2 - 3$

$g(1) = 4$

(b) $(g \circ f)(1) = g(f(1)) = g(-1) = 4 \cdot (-1) = -4$

$f(x) = 2x^2 - 3$ $g(x) = 4x$

$f(1) = -1$

(c) $(f \circ f)(-2) = f(f(-2)) = f(5) = 2 \cdot 5^2 - 3 = 47$

$f(-2) = 2(-2)^2 - 3 = 5$

(d) $(g \circ g)(-1) = g(g(-1)) = g(-4) = 4 \cdot (-4) = -16$

$g(-1) = -4$

Finding a Composite Function and Its Domain

Suppose that $f(x) = x^2 + 3x - 1$ and $g(x) = 2x + 3$.

Find: (a) $f \circ g$ (b) $g \circ f$

Then find the domain of each composite function.

The domain of f and the domain of g are the set of all real numbers.

(a) $(f \circ g)(x) = f(g(x)) = f(2x + 3) = (2x + 3)^2 + 3(2x + 3) - 1$

$f(x) = x^2 + 3x - 1$

$= 4x^2 + 12x + 9 + 6x + 9 - 1 = 4x^2 + 18x + 17$

Since the domains of both f and g are the set of all real numbers, the domain of $f \circ g$ is the set of all real numbers.

*Consult your owner's manual for the appropriate keystrokes.

AMPLE 4 Finding a Composite Function and Its Domain

Suppose that $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{4}{x-1}$.

Find: (a) $f \circ g$ (b) $f \circ f$

Then find the domain of each composite function.

Solution The domain of f is $\{x \mid x \neq -2\}$ and the domain of g is $\{x \mid x \neq 1\}$.

(a) $$(f \circ g)(x) = f(g(x)) = f\left(\frac{4}{x-1}\right) = \frac{1}{\frac{4}{x-1} + 2} = \frac{x-1}{4 + 2(x-1)} = \frac{x-1}{2x+2} = \frac{x-1}{2(x+1)}$$

$f(x) = \frac{1}{x+2}$ Multiply by $\frac{x-1}{x-1}$.

the domain of $f \circ g$ to be $\{x \mid x \neq -1, x \neq 1\}$.

(b) $$(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x+2}\right) = \frac{1}{\frac{1}{x+2} + 2} = \frac{x+2}{1 + 2(x+2)} = \frac{x+2}{2x+5}$$

$f(x) = \frac{1}{x+2}$ Multiply by $\frac{x+2}{x+2}$.

The domain of $f \circ f$ consists of those x in the domain of f, $\{x \mid x \neq -2\}$, for which

$$f(x) = \frac{1}{x+2} \neq -2$$

$$\frac{1}{x+2} = -2$$
$$1 = -2(x+2)$$
$$1 = -2x - 4$$
$$2x = -5$$
$$x = -\frac{5}{2}$$

or, equivalently,

$$x \neq -\frac{5}{2}$$

The domain of $f \circ f$ is $\left\{x \middle| x \neq -\frac{5}{2}, x \neq -2\right\}$.

We could also find the domain of $f \circ f$ by recognizing that -2 is not in the domain of f and so should be excluded from the domain of $f \circ f$. Then, looking at $f \circ f$, we see that x cannot equal $-\frac{5}{2}$. Do you see why? Therefore, the domain of $f \circ f$ is $\left\{x \middle| x \neq -\frac{5}{2}, x \neq -2\right\}$.