Sum and difference Identities

Finding the exact value under the given conditions:

$\sin(a+b) = \sin a \cos b + \cos a \sin b$ 1

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

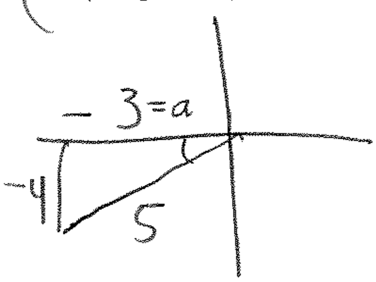
$\sin(a+b) = \sin a \cos b + \cos a \sin b$ ②

If $\sin a = -\frac{4}{5}$, in Quad $\underline{III}$

$\cos b = \frac{12}{13}$, in Quad $\underline{IV}$

Find the exact value of $\sin(a+b)$

ⓐ



son cah toa

$s = \frac{o}{h} = \frac{-4}{5}$ $c = \frac{a}{h} = \frac{-3}{5}$

$a^2 + b^2 = c^2$

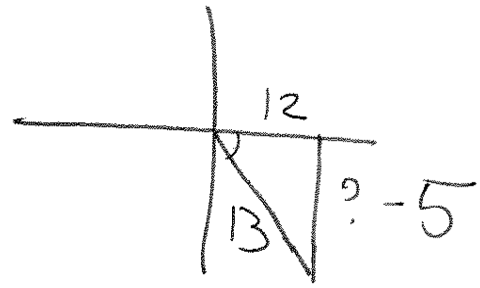
$(-4)^2 = (5)^2$

$16 = 25$

$a^2 = 9$

$a = \pm 3$

ⓑ



son cah toa.

$s = \frac{o}{h} = \frac{-5}{13}$ $c = \frac{a}{h} = \frac{12}{13}$

$a^2 + b^2 = c^2$

$(12)^2 \quad (13)^2$

$144 + b^2 = 169$

$b^2 = 25$

$b = \pm 5$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$= \left(-\frac{4}{5}\right)\left(\frac{12}{13}\right) + \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{-48}{65} + \frac{+15}{65}$

$\sin(a+b) = \boxed{\frac{-33}{65}}$

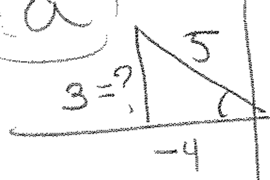
$\sin(a-b) = \sin a \cos b - \cos a \sin b$ (3)

If $\cos a = \frac{-4}{5}$ is in $\frac{\pi}{2} < a < \pi$

$\cos b = \frac{12}{13}$ is in $\frac{3\pi}{2} < a < 2\pi$

Find the exact value of $\sin(a-b)$

a



$x^2+b^2=c^2$

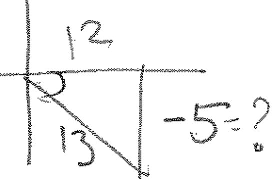
$16 = 25$

$x^2 = 9$

$a = 3$

son	cah	toa
$\frac{3}{5}$	$\frac{-4}{5}$	$\frac{3}{-4}$

b



$a^2+b^2=c^2$

$144 = 169$

$a^2 = 25$

$a = \pm 5$

son	cah	toa
$\frac{-5}{13}$	$\frac{12}{13}$	$\frac{-5}{12}$

$\sin(a-b) = \sin a \cos b - \cos(a) \sin b$

$$\left(\frac{3}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$$

$$\frac{36}{65} - \frac{+20}{65}$$

$$\sin(a-b) = \boxed{\frac{16}{65}}$$

(4)

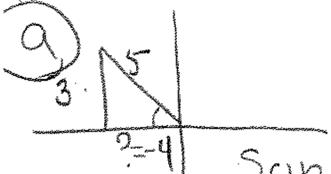
$\cos(a+b) = \cos a \cos b - \sin a \sin b$

If $\sin a = 3/5$, is in Quad II

$\cos B = \frac{12}{13}$, is in $0 < a < \frac{\pi}{2}$

Find the exact value of $\cos(a+b)$

(a)



Soh cah toa

$a^2+b^2=c^2$

$(3)^2=(5)^2$

$9 = 25$

$-9 \quad -9$

$a^2 = 16$

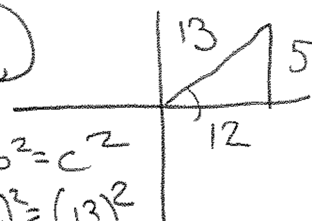
$a = \pm 4$

$s = \frac{o}{h} = \frac{3}{5}$

$c = \frac{a}{h} = \frac{-4}{5}$

$t = \frac{o}{a} = \frac{3}{-4}$

(B)



soh cah toa

$s = \frac{o}{h} = \frac{5}{13}$ $c = \frac{a}{h} = \frac{12}{13}$ $t = \frac{o}{a} = \frac{5}{12}$

$a^2+b^2=c^2$

$(12)^2=(13)^2$

$144 = 169$

$-144 \quad -144$

$a^2 = 25$

$a = \pm 5$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\left(-\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$

$-\frac{48}{65} - \frac{15}{65}$

$\cos(a+b) = \boxed{\frac{-63}{65}}$

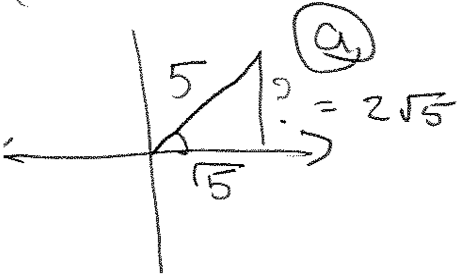
$\cos(a-b) = \cos a \cos b + \sin a \sin b$

If $\cos a = \dfrac{\sqrt{5}}{5}$, $0 < a < \dfrac{\pi}{2}$

$\sin b = \dfrac{-4}{5}$, $-\dfrac{\pi}{2} < b < 0$

find the exact value of $\cos(a-b)$

(a)



son can toa

$s = \dfrac{o}{h} = \dfrac{2\sqrt{5}}{5}$ $c = \dfrac{a}{h} = \dfrac{\sqrt{5}}{5}$

$a^2 + b^2 = c^2$

$(\sqrt{5})^2 = (5)^2$

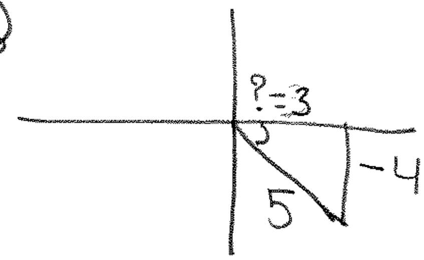
$5 = 25$

$a^2 = 20$

$a = \sqrt{20} \Rightarrow \sqrt{4} \cdot \sqrt{5}$

$a = 2\sqrt{5}$

(b)



son can toa

$s = \dfrac{o}{h} = \dfrac{-4}{5}$ $c = \dfrac{a}{h} = \dfrac{3}{5}$

$a^2 + b^2 = c^2$

$(-4)^2 = (5)^2$

$16 = 25$

$a^2 = 9$

$a = 3$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\left(\dfrac{\sqrt{5}}{5}\right)\left(\dfrac{3}{5}\right) + \left(\dfrac{2\sqrt{5}}{5}\right)\left(\dfrac{-4}{5}\right)$

$= \dfrac{3\sqrt{5}}{25} + -\dfrac{8\sqrt{5}}{25}$

$= \dfrac{-5\sqrt{5}}{25}$

$\cos(a-b) = \boxed{\dfrac{-\sqrt{5}}{5}}$

6

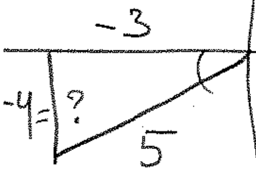
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

If $\cos a = \frac{-3}{5}$, in $\pi \ a < \frac{3\pi}{2}$

$\sin b = \frac{-5}{13}$, in $\frac{3\pi}{2} \ a < 2\pi$

find the exact value of tan (a+b)

a)



son	cah	toa
$\frac{-4}{5}$	$\frac{-3}{5}$	$\frac{-4}{-3}$

$a^2+b^2=c^2$

$9 = 25$

$a^2 = 16$

$a = \pm 4$

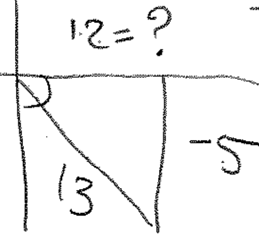
B)

$a^2+b^2=c^2$

$25 = 169$

$a^2 = 144$

$a = \pm 12$



son	cah	toa
$\frac{-5}{13}$	$\frac{12}{13}$	$\frac{-5}{12}$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \Rightarrow \frac{\left(\left(\frac{4}{3}\right) + \left(\frac{-5}{12}\right)\right)}{\left(1 + \left(\frac{4}{3}\right)\left(\frac{-5}{12}\right)\right)}$$

$$= \frac{\left(\frac{4}{4}\left(\frac{4}{3}\right) + \left(-\frac{5}{12}\right)\right)}{\frac{36}{36}\left(\frac{1}{1}\right) + \left(-\frac{20}{36}\right)} \Rightarrow \frac{\left(\frac{16}{12} - \frac{5}{12}\right)}{\left(\frac{36}{36} - \frac{20}{36}\right)} \Rightarrow \frac{\frac{11}{12}}{\frac{16}{36}}$$

$$\frac{11}{12} \cdot \frac{36}{16} \Rightarrow \frac{11}{1} \cdot \frac{3}{16} = \boxed{\frac{33}{16}} \Rightarrow \tan(a+b)$$

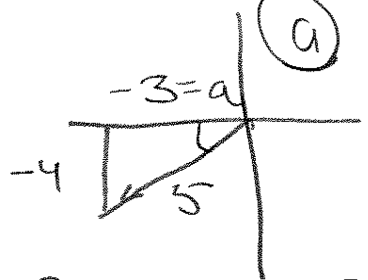
17

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

If $\sin a = -\frac{4}{5}$ in Quad III,
$\cos b = \frac{12}{13}$ in Quad IV

find the exact value of $\tan(a-b)$

a



son
cah
toa

$a^2 + b^2 = c^2$

$(-4)^2 = (5)^2$

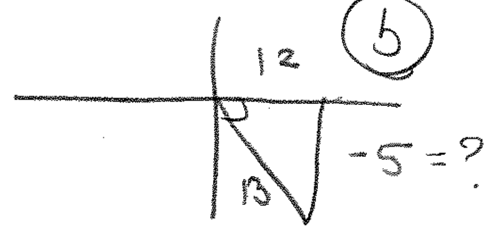
$16 = 25$

$a^2 = 9$

$a = \pm 3$

$t = \frac{o}{a} = \frac{-4}{-3} = \frac{4}{3}$

b



son
cah
toa

$a^2 + b^2 = c^2$

$(12)^2 \qquad (13)^2$

$144 + b^2 = 169$

$b^2 = 25$

$b = \pm 5$

$t = \frac{o}{a} = \frac{-5}{12}$

$$= \frac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}$$

$$= \frac{\left(\left(\frac{4}{3}\right) - \left(\frac{-5}{12}\right)\right)}{\left(1 + \left(\frac{4}{3}\right)\left(\frac{-5}{12}\right)\right)} = \frac{\frac{12}{12}\left(\frac{4}{3}\right) - \left(\frac{-5}{12}\right)\frac{3}{3}}{\frac{1}{1} + \left(-\frac{20}{36}\right)}$$

$$= \frac{\frac{48}{36} + \frac{15}{36} = \frac{63}{36}}{\frac{36}{36}\left(\frac{1}{1}\right) - \frac{20}{36}} \Rightarrow \frac{\frac{63}{36}}{\frac{16}{36}} \Rightarrow$$

$$= \frac{63}{36} \cdot \frac{36}{16} \Rightarrow \boxed{\frac{63}{16}} = \tan(a-b)$$